

Compact Version

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [30 Points] Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x \rightarrow 0} \frac{xe^x - x}{x \sin(3x)}$

(b) $\lim_{x \rightarrow \infty} \left(1 - \frac{3}{x}\right)^x$

(c) $\lim_{x \rightarrow \infty} (e^x + x)^{\frac{1}{x}}$

2. [30 Points] Compute each of the following **definite integrals**. Please simplify your answer.

(a) $\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} dx$

(b) $\int_0^{\ln 7} \tanh x dx$

(c) $\int_1^e (\ln x)^2 dx$

3. [40 Points] Compute each of the following **indefinite integrals**.

(a) $\int \left(x + \frac{1}{e^{3x}}\right)^2 dx$

(b) $\int x \arctan x dx$

(c) $\int \frac{1}{(x^2 + 4)^{\frac{5}{2}}} dx$

(d) $\int \frac{x^2}{\sqrt{4 - x^2}} dx$

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\int \frac{1}{\sqrt{e^x - 1}} dx$

OPTIONAL BONUS #2 Compute the following **indefinite integral**.

2. $\int \frac{xe^x}{\sqrt{1 + e^x}} dx$