

Math 121 Midterm Exam #1 September 24, 2021

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. Limits [25 Points total] Evaluate each of the following. Please justify/simplify. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) Show that $\lim_{x \rightarrow 0} \frac{\ln(1-5x) + \arcsin(5x)}{3xe^x - \arctan(3x)} = \boxed{-\frac{25}{6}}$

(b) Show that $\lim_{x \rightarrow 0^+} x^5 \ln x = \boxed{0}$

(c) Show that $\lim_{x \rightarrow \infty} \left(1 - \arcsin\left(\frac{2}{x^4}\right)\right)^{x^4} = \boxed{e^{-2}}$

Integrals [75 Points total, 15 points each] Compute each of the following integrals. Please justify/simplify.

2. Show that $\int_e^{e^3} \frac{1}{x[3 + (\ln x)^2]} dx = \boxed{\frac{\pi}{6\sqrt{3}}}$

3. Compute $\int \frac{1}{(4-x^2)^{\frac{3}{2}}} dx$

4. Compute $\int x^2 \arcsin x dx$

5. Show that $\int_1^{e^2} \sqrt{x} \cdot \ln x dx = \boxed{\frac{8e^3 + 4}{9}}$

6. Compute $\int \frac{1}{(x^2 + 4)^2} dx \stackrel{\text{hint}}{=} \int \frac{1}{(\sqrt{x^2 + 4})^4} dx$