

Math 121 Midterm Exam #1 October 2, 2019

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [14 Points]

(a) Use implicit differentiation to **PROVE** that $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$.

(b) From part (a) we now know that $\int \frac{1}{1+x^2} dx = \arctan x + C$. You may use this fact to **PROVE** that

$$\int \frac{1}{3+x^2} dx = \frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right) + C \quad \leftarrow \text{Prove this.}$$

2. [24 Points] Evaluate the following **limit**. Please justify your answer. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist. Simplify.

(a) $\lim_{x \rightarrow 0} \frac{\arcsin x + x^2 + \ln(1-x)}{\cosh(2x) - \arctan(5x) - e^{-5x}}$

(b) $\lim_{x \rightarrow \infty} \left[1 - \arcsin\left(\frac{5}{x^2}\right)\right]^{x^2}$

3. [42 Points] Compute each of the following **integrals**. Please simplify your answer.

(a) $\int_2^{2\sqrt{3}} \frac{1}{\sqrt{16-x^2}} dx$

(b) $\int \frac{x^2}{\sqrt{16-x^2}} dx$

(c) $\int_{\frac{\pi}{2}}^{\pi} \frac{\cos x}{3 + \sin^2 x} dx$

(d) $\int \frac{1}{x[3 + (\ln x)^2]^{\frac{3}{2}}} dx$

4. [20 Points] Compute each of the following **integrals**. Please simplify your answer.

(a) Show that $\int_0^1 x^2 \arcsin x \, dx = \frac{\pi}{6} - \frac{2}{9}$

(b) Show that $\int_0^1 (x+1) \arctan x \, dx = \frac{\pi - 1 - \ln 2}{2}$

OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\int \frac{1}{x^{\frac{3}{2}}(1+x^{\frac{1}{3}})} \, dx$