

Math 121 Midterm Exam #1
October 3, 2018

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [8 Points] Use implicit differentiation to **PROVE** that $\frac{d}{dx} \arctan(5x) = \frac{5}{1 + 25x^2}$

2. [24 Points] Evaluate each of the following **limits**. Please justify your answer. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist. Simplify.

(a) $\lim_{x \rightarrow 0} \frac{\arcsin x + x^2 - x}{\cosh(2x) - \arctan(5x) - e^{-5x}}$

(b) $\lim_{x \rightarrow \infty} \left[1 + \ln\left(1 - \frac{5}{x}\right)\right]^x$

3. [48 Points] Compute each of the following **integrals**. Please simplify your answer.

(a) $\int_2^{2\sqrt{3}} \frac{1}{\sqrt{16 - x^2}} dx$

(b) $\int \frac{x^2}{\sqrt{16 - x^2}} dx$

(c) $\int_e^{e^3} \frac{1}{x[3 + (\ln x)^2]} dx$

(d) $\int \frac{1}{x[3 + (\ln x)^2]^{\frac{3}{2}}} dx$

4. [20 Points] Compute the following **integrals**. Please simplify your answer.

(a) Show that $\int_0^1 x^2 \arcsin x \, dx = \frac{\pi}{6} - \frac{2}{9}$

(b) $\int \ln(x^2 + 7) \, dx$

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\int \frac{x^3 - 9x^2 + 27x - 27}{(x^2 - 6x + 13)^{\frac{5}{2}}} \, dx$

OPTIONAL BONUS #2 Compute the following **indefinite integral**.

2. $\int \frac{1}{x^{\frac{3}{2}} \left(1 + x^{\frac{1}{3}}\right)} \, dx$