Math 121   Midterm Exam #1   October 4, 2017

● This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

● You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\dfrac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

● Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [12 Points]

(a) Use implicit differentiation to **PROVE** that $\dfrac{d}{dx}\arcsin x = \dfrac{1}{\sqrt{1-x^2}}$.

(b) Use implicit differentiation to **PROVE** that $\dfrac{d}{dx}\ln x = \dfrac{1}{x}$.

(c) Use implicit differentiation to **PROVE** that $\dfrac{d}{dx}\sinh^{-1} x = \dfrac{1}{\sqrt{1+x^2}}$.

2. [30 Points]   Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to 0} \dfrac{\ln(1-x) + \sinh x}{\arctan(2x) - e^{2x} + 1}$

(b) $\displaystyle\lim_{x\to\infty} (\ln x)^{\frac{6}{x}}$

(c) $\displaystyle\lim_{x\to\infty} \left[1 - \arctan\left(\dfrac{3}{x^2}\right)\right]^{x^2}$

3. [28 Points]   Compute the following **definite integrals**. Please simplify your answer.

(a) Show that $\displaystyle\int_1^{e^4} \dfrac{\ln x}{\sqrt{x}}\, dx = 4e^2 + 4$

(b) $\displaystyle\int_2^{2\sqrt{3}} \dfrac{1}{\sqrt{16 - x^2}}\, dx$

(c) $\displaystyle\int_0^{\ln 3} \dfrac{e^x}{3 + e^{2x}}\, dx$

4. [30 Points]   Compute the following **indefinite integrals**.

(a) $\displaystyle\int x \arcsin x \; dx$

(b) $\displaystyle\int \frac{1}{(x^2 + 4)^{\frac{7}{2}}} \; dx$

(c) $\displaystyle\int \ln(x^2 + 7) \; dx$

****

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

****

OPTIONAL BONUS #1   Compute the following **indefinite integral**.

1. $\displaystyle\int \frac{1}{(x^4 + 4x^3 + 6x^2 + 4x + 1)\sqrt{x^2 + 2x - 8}} \; dx$

OPTIONAL BONUS #2   Compute the following **indefinite integral**.

2. $\displaystyle\int \frac{\arcsin x}{x^2} \; dx$