Math 121   Midterm Exam #1   October 5, 2016

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $\arcsin\left(\frac{1}{2}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [10 Points]   Use implicit differentiation to **PROVE** that $\dfrac{d}{dx}\arcsin(3x) = \dfrac{3}{\sqrt{1-9x^2}}$.

2. [30 Points]   Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to 0^+}\ (1-3\sin x)^{\frac{1}{x}}$

(b) $\displaystyle\lim_{x\to 0}\ \frac{\arcsin x\ +\cos(3x)-e^x}{\arctan(3x)\ +x^2\ -\sinh(3x)}$

(c) $\displaystyle\lim_{x\to\infty}\ \left(1-\arcsin\left(\frac{5}{x^4}\right)\right)^{3x^4}$

3. [45 Points]   Compute the following **definite integral**. Please simplify your answer.

(a) $\displaystyle\int_0^{\sqrt{3}} x\arctan x\ dx$

(b) $\displaystyle\int_2^{2\sqrt{3}} \frac{x^2}{\sqrt{16-x^2}}\ dx$

(c) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\cos x}{9+4\sin^2 x}\ dx$

(d) $\displaystyle\int_1^e \left[\ln(x^3)\right]^2\ dx$

4. [15 Points]   Compute the following **indefinite integral**.

$$\int \frac{\cos x}{\left(1 + \sin^2 x\right)^{\frac{7}{2}}} \, dx$$

OPTIONAL BONUS #1   Compute the following **indefinite integral**.

1. $\displaystyle\int \frac{x^4 - 8x^3 + 24x^2 - 32x + 16}{(4x - x^2)^{\frac{7}{2}}} \, dx$