Math 121   Midterm Exam #1   October 2, 2015

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [8 Points]   Use implicit differentiation to **PROVE** that $\dfrac{d}{dx}\cosh^{-1}(3x) = \dfrac{3}{\sqrt{9x^2 - 1}}$.

2. [32 Points]   Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to\infty} x\left(\frac{\pi}{2} - \arctan(4x)\right)$

(b) $\displaystyle\lim_{x\to\ln 4} \frac{\sinh(x - \ln 4)}{e^{-2x} - \dfrac{1}{16}}$

(c) $\displaystyle\lim_{x\to 0} \left(1 + \ln(1 - 3x)\right)^{\frac{1}{x}}$

(d) $\displaystyle\lim_{x\to\infty} \left(1 - \arcsin\left(\frac{6}{x}\right)\right)^x$

3. [40 Points]   Compute the following **definite integrals**. Please simplify your answer.

(a) $\displaystyle\int_0^1 x\arctan x \; dx$

(b) $\displaystyle\int_1^{\sqrt{3}} \frac{x^2}{\sqrt{4 - x^2}} \; dx$

(c) $\displaystyle\int_{-1}^0 x^3 \sqrt{1 - x^2} \; dx$ using a **trigonometric substitution**.

(d) $\displaystyle\int_1^{\sqrt{e}} \left[\ln(x^2)\right]^2 \; dx$

4. [20 Points]   Compute the following **indefinite integrals**.

(a) $\displaystyle \int \frac{1}{(1+x^2)(5+(\arctan x)^2)}\, dx$

(b) $\displaystyle \int \frac{1}{(4+x^2)^{\frac{7}{2}}}\, dx$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1   Compute the following **indefinite integral**.

1. $\displaystyle \int \frac{e^{\arcsin x}\, x^2}{\sqrt{1-x^2}}\, dx$