Math 121 Midterm Exam #1 October 1, 2014

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [10 Points]

(a) Use implicit differentiation to **PROVE** that $\dfrac{d}{dx}\sinh^{-1} x = \dfrac{1}{\sqrt{1+x^2}}$.

(b) From part (a) we now know that $\displaystyle\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1} x + C$. You may use this fact to **PROVE** that

$$\int \frac{1}{\sqrt{7+x^2}}\,dx = \sinh^{-1}\left(\frac{x}{\sqrt{7}}\right) + C \quad \longleftarrow \textbf{Prove this.}$$

2. [30 Points] Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to 0} \frac{\sin^{-1} x - \arctan x + x^2}{\sinh^{-1}(3x) + e^{-3x} - 1}$

(b) $\displaystyle\lim_{x\to\infty} \left(\sqrt{1-\frac{3}{x}} - \sinh\left(\frac{1}{x}\right)\right)^x$

(c) **Show** that $\displaystyle\lim_{x\to\infty} \left(x^2+1\right)^{\frac{\ln 3}{\ln x}} = 9.$

3. [40 Points] Compute the following **definite integrals**. Please simplify your answer.

(a) $\displaystyle\int_0^{\ln 5} \sinh x\,dx$

(b) $\displaystyle\int_1^{\sqrt{3}} \frac{x^2}{\sqrt{4-x^2}}\,dx$

(c) $\displaystyle\int_{-1}^{0} x^3\sqrt{1-x^2}\,dx$ using a **trigonometric substitution**.

(d) $\displaystyle\int_1^{\sqrt{e}} (\ln(x^2))^2 \; dx$

4. [20 Points] Compute the following **indefinite integrals**.

(a) $\displaystyle\int \frac{1}{(1+x^2)(5+(\arctan x)^2)} \; dx$

(b) $\displaystyle\int \frac{\cos x}{\left(9+\sin^2 x\right)^{\frac{3}{2}}} \; dx$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\displaystyle\int \frac{xe^x}{\sqrt{1+e^x}} \; dx$