

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- You need *not* simplify algebraically complicated answers. However, numerical answers such as  $\sin\left(\frac{\pi}{6}\right)$ ,  $4^{\frac{3}{2}}$ ,  $\sinh(\ln 3)$ ,  $e^{\ln 4}$ ,  $\ln(e^7)$ , or  $e^{3\ln 3}$  should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

**1.** [10 Points]

(a) Let  $y = \arcsin x$ . Use implicit differentiation to **PROVE** that  $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ .

(b) From part (a) we now know that  $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$ . You may use this fact to **PROVE**, for a constant  $a > 0$ , that

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C \quad \leftarrow \text{Prove this.}$$

**2.** [30 Points] Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value,  $+\infty$  or  $-\infty$ , or Does Not Exist.

(a)  $\lim_{x \rightarrow 0} \frac{\ln(1-x) + \sinh x}{\arctan(2x) - e^{2x} + 1}$

(b)  $\lim_{x \rightarrow \infty} \left(e^{\frac{1}{x}} - \frac{4}{x}\right)^x$

(c)  $\lim_{x \rightarrow \infty} (x^2 + 1)^{\frac{1}{\ln x}}$

**3.** [30 Points] Compute the following **definite integral**. Please simplify your answer.

(a)  $\int_0^{\ln 7} x \sinh x \, dx$

(b)  $\int_3^{3\sqrt{3}} \frac{1}{\sqrt{36-x^2}} + \frac{1}{9+x^2} \, dx$

(c)  $\int_0^1 \ln(x^2+1) \, dx$

**4.** [30 Points] Compute each of the following **indefinite integral**.

(a)  $\int x \arcsin x \, dx$

(b)  $\int \frac{e^x}{(e^{2x}+4)^{\frac{7}{2}}} \, dx$

(c)  $\int \frac{\cos x}{1+\sin^2 x} \, dx$

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## OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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**OPTIONAL BONUS #1** Compute the following **indefinite integral**.

1.  $\int e^{\sqrt{1+\sqrt{x}}} \, dx$

**OPTIONAL BONUS #2** Compute the following **indefinite integral**.

2.  $\int \frac{\ln(x-1)}{\sqrt{x}} \, dx$