

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [6 Points] Let $y = \arcsin x$. Use implicit differentiation to **PROVE** that $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$.

2. [24 Points] Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x \rightarrow 0} \frac{\arctan(3x) - \sinh(3x)}{\ln(1+x) - x}$

(b) $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x^2}\right)^{7x^2}$

(c) $\lim_{x \rightarrow \infty} (\ln x)^{\frac{3}{x}}$

3. [30 Points] Compute the following **definite integral**. Please simplify your answer.

(a) $\int_0^{\ln 7} \sinh x \, dx$

(b) $\int_3^{3\sqrt{3}} \frac{1}{\sqrt{36-x^2}} + \frac{1}{9+x^2} \, dx$

(c) $\int_1^{\sqrt{e}} \ln(x^2) \, dx$

4. [40 Points] Compute the following **indefinite integral**.

(a) $\int x \arcsin x \, dx$

(b) $\int \frac{1}{(x^2 + 4)^{\frac{7}{2}}} \, dx$

(c) $\int \frac{e^x}{1 + e^{2x}} \, dx$

(d) $\int x^2 e^{-7x} \, dx$

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\int \frac{x^3}{1 - \sin(x^2)} \, dx$

OPTIONAL BONUS #2 Compute the following **indefinite integral**.

2. $\int \frac{1}{1 + 3 \sin^2 \theta} \, d\theta$