Name:_____

Amherst College

DEPARTMENT OF MATHEMATICS

Math 121

Midterm Exam #1

September 30, 2011

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\dfrac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		30
2		30
3		40
Total		100

1. [30 Points]   Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim\limits_{x\to 0} \dfrac{1 - \cosh(2x)}{x + \ln(1 - x)}$

(b) $\lim\limits_{x\to\infty} \dfrac{\arctan x}{\sinh\left(\frac{1}{x}\right)}$

1. (Continued)   Evaluate the following **limit**. Please justify your answer. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(c) $\displaystyle\lim_{x\to\infty} \left(x^2+1\right)^{\frac{1}{\ln x}}$

(d) $\displaystyle\lim_{x\to\infty} \left(e^{\frac{1}{x}} - \frac{4}{x}\right)^x$

2. [30 Points]   Compute each of the following **definite integrals**. Please simplify your answer.

(a) $\displaystyle \int_1^{\sqrt{3}} \frac{x+1}{\sqrt{4-x^2}}\, dx$

(b) $\displaystyle \int_0^{\ln 7} x\, \sinh x\, dx$

2. (Continued)   Compute the following **definite integral**. Please simplify your answer.

(c) $\displaystyle\int_0^1 \frac{y^2}{e^{2y}}\, dy$

3. [40 Points] Compute each of the following **indefinite integrals**.

(a) $\int \sec x + \tan x\ dx$

(b) $\int x\ \arcsin x\ dx$

3. (Continued)   Compute the following **indefinite integral**.

(c) $\displaystyle\int \frac{1}{(x^2 + 4)^{\frac{5}{2}}}\ dx$

3. (Continued)   Compute the following **indefinite integral**.

(d) $\int \ln(x^2 + 1)\ dx$

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1   Compute the following **indefinite integral**.

1. $\displaystyle \int \frac{1}{1 - \sin x}\ dx$

OPTIONAL BONUS #2   Compute the following **indefinite integral**.

2. $\displaystyle \int e^{\sqrt{1 + \sqrt{x}}}\ dx$

OPTIONAL BONUS #3   Compute the following **indefinite integral**.

3. $\displaystyle \int \frac{\ln(x - 1)}{\sqrt{x}}\ dx$

OPTIONAL BONUS #4   Compute the following **indefinite integral**.

4. $\displaystyle \int x \cos^6 x\ dx$