Math 121 Final May 13, 2026

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, $e^{3\ln 3}$, or $\arctan(\sqrt{3})$, should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [15 Points] (a) Use Series to show that $\displaystyle\lim_{x\to 0} \frac{5x - \ln(1+5x)}{1 + 3x - e^{3x} + 2x - \arctan(2x)} = \boxed{-\frac{25}{9}}$

(b) Compute $\displaystyle\lim_{x\to 0} \frac{5x - \ln(1+5x)}{1 + 3x - e^{3x} + 2x - \arctan(2x)}$ again using L'Hôpital's Rule.

2. [16 Points] Compute the following integral. Simplify.

(a) $\displaystyle\int \sqrt{9 - x^2}\, dx$ (b) $\displaystyle\int x^4 \arcsin x \, dx$

3. [24 Points] For each of the following **Improper** integrals, determine whether it Converges or Diverges. If it converges, find its value. Simplify.

(a) $\displaystyle\int_0^e \frac{\ln x}{\sqrt{x}}\, dx$ (b) $\displaystyle\int_{-\infty}^0 \frac{1}{x^2 - 4x + 8}\, dx$ (c) $\displaystyle\int_{-4}^3 \frac{20 - x}{x^2 - 4x - 32}\, dx$

4. [24 Points] Find the **Sum** of each of the following series (which do converge). Simplify.

(a) $\displaystyle\frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \ldots$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\, 2^{n+1}\, (\ln 5)^n}{n!}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\, \pi^{2n}}{(36)^n\, (2n+1)!}$

(d) $\displaystyle -\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots$ (e) $\displaystyle -1 + \frac{\pi^2}{2!} - \frac{\pi^4}{4!} + \frac{\pi^6}{6!} - \ldots$ (f) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{e^{3n}}$

5. [20 Points] (a) Find the Interval/Radius of Convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (5x+8)^n}{(5n+8)^3\ \ 2^n}$

(b) **CREATE** a Power Series centered at $a = 5$ which has a Radius of Convergence, $R = \infty$. Once you create your series, then proceed to justify that the Interval of Convergence is indeed $I = (-\infty, \infty)$.

6. [18 Points] Determine whether each series **Converges** or **Diverges**. Name any Convergence Test(s) you use; justify all of your work.

(a) $-\frac{3}{\sqrt{1}} - \frac{3}{\sqrt{2}} - \frac{3}{\sqrt{3}} - \frac{3}{\sqrt{4}} - \frac{3}{\sqrt{5}} - \dots$ (b) $\sum_{n=1}^{\infty} \left(\frac{1}{\ln(2026)} \right)^n$

(c) **CREATE** a Series that Diverges by the n^{th} Term Divergence Test AND requires L'Hôpital's Rule in the Divergence Test's Limit.

7. [10 Points] Use the **Absolute Convergence Test** to show that $\sum_{n=1}^{\infty} (-1)^n \frac{n}{e^n}$ **Converges**. You are required to use the **Integral Test** on the Absolute Series.

8. [24 Points] Determine whether the series is **Absolutely Convergent** or **Conditionally Convergent** or **Divergent**. Name any Convergence Test(s) you use, justify all of your work.

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{n^5 + 8}{n^8 + 5}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^n}$

(c) **CREATE** an Alternating series which is **Conditionally Convergent**. Continue on to justify that this series is Conditionally Convergent. Keep the choice simple.

9. [10 Points] Use Series to Estimate $\int_0^{\frac{1}{2}} x^2 e^{-x}\, dx$ with error less than $\frac{1}{200}$. Simplify.

Tip: common denominator for 24 and 64 is 192, and $24 \cdot 8 = 192$ and $64 \cdot 3 = 192$ and $2^5 = 32$

10. [15 Points] Answer in Sigma $\sum_{n=0}^{\infty}$ notation.

(a) Find the MacLaurin Series for $\ln(8 + x^3)$. Hint: $\ln(8 + x^3) = \int \frac{3x^2}{8 + x^3}\, dx = \dots$.

Solve for C and State the Radius of Convergence.

(b) Demonstrate **TWO** Different methods for deriving the MacLaurin Series for $\cos x$.

You may use the MacLaurin Series formula for $\sin x$ *without extra justification.*

11. [24 Points] For **each** of the following problems, do the following **THREE** things:

1. **Sketch** the Polar curve(s) and **Shade** the described bounded region.

2. Set-Up but **DO NOT EVALUATE** an Integral representing the area of the described bounded region.

3. Set-Up but **DO NOT EVALUATE** another **slightly different** Integral representing the same area of the described bounded region.

(a) The **Area** bounded Inside $r = 2 - 2\sin\theta$

(b) The **Area** bounded Outside the polar curve $r = 1 + \sin\theta$ and Inside $r = 3\sin\theta$.

(c) The **Area** bounded Inside **Both** of the curves $r = 3 + 3\cos\theta$ and $r = 3 - 3\cos\theta$.

(d) The **Area** bounded Outside the polar curve $r = 2 - 2\cos\theta$ and Inside $r = -6\cos\theta$.