Math 121 Final Exam May 9, 2017

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, $e^{3\ln 3}$, $\arctan(\sqrt{3})$, or $\cosh(\ln 3)$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [12 Points] Evaluate the following **limit**. Please justify your answer. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist. Simplify.

(a) $\lim\limits_{x\to 0} \dfrac{5xe^x - \arctan(5x)}{\sinh x + \ln(1-x)}$ (b) Compute $\lim\limits_{x\to\infty} \left(1 - \arcsin\left(\dfrac{5}{x}\right)\right)^x$

2. [16 Points] Evaluate each of the following integrals.

(a) $\displaystyle\int \frac{\cos x}{\left(1+\sin^2 x\right)^{\frac{7}{2}}}\,dx$ (b) $\displaystyle\int_{-1}^{0} x \arcsin x\,dx$

3. [40 Points] For each of the following **improper integrals**, determine whether it converges or diverges. If it converges, find its value. Simplify.

(a) $\displaystyle\int_0^{e^3} \frac{1}{x\left[9+(\ln x)^2\right]}\,dx$ (b) $\displaystyle\int_0^{e} \frac{\ln x}{\sqrt{x}}\,dx$

(c) $\displaystyle\int_1^{2} \frac{2}{x^2-6x+8}\,dx$ (d) $\displaystyle\int_5^{\infty} \frac{1}{x^2-6x+13}\,dx$

4. [20 Points] Find the **sum** of each of the following series (which do converge). Simplify.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 3^{2n-1}}{4^{2n+1}}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\, (\ln 8)^n}{3^{n+1}\, n!}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(36)^n\, (2n+1)!}$

(d) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots$ (e) $-\frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \frac{\pi^8}{8!} - \ldots$ (f) $-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \ldots$

5. [30 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **divergent**. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, (n^3+7)}{n^7+3}$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\, \frac{n+1}{n^2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \arctan(n^2)}{n^2+1}$ (d) $\displaystyle\sum_{n=1}^{\infty} \arctan\left(\frac{n^2}{n^2+1}\right)$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, (3n)!\, \ln n}{(n!)^2\, 2^{4n}\, n^n}$

6. [12 Points] Find the **Interval** and **Radius** of Convergence for the following power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,(3x+1)^n}{(n+7)\cdot 7^n}$. Analyze carefully and with full justification.

7. [10 Points] (a) Use MacLaurin series to **Estimate** $\displaystyle\int_0^1 x\sin\left(x^2\right)\,dx$ with error less than $\dfrac{1}{1000}$. Please analyze with detail and justify carefully. Simplify.

(b) Estimate $\dfrac{1}{\sqrt{e}}$ with error less than $\dfrac{1}{100}$. Justify in words that your error is indeed less than $\dfrac{1}{100}$.

8. [8 Points] For each of the following functions, find the MacLaurin Series and, then **State** the Radius of Convergence.

(a) $f(x) = \sinh x$ (b) $f(x) = \dfrac{1}{(1-x)^2}$. Hint: Differentiate $\left(\dfrac{1}{1-x}\right)$

9. [18 Points]

(a) Consider the region bounded by $y = \arcsin x$, $y = \dfrac{\pi}{2}$, and $x = 0$. Rotate the region about the vertical line $\boxed{x = 3}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

(b) Consider the region bounded by $y = \arctan x$, $y = 0$, $x = 0$ and $x = 1$. Rotate the region about the vertical line $\boxed{y\text{-axis}}$. **COMPUTE** the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

10. [14 Points]

Consider the Parametric Curve represented by $x = \ln t + \ln(1-t^2)$ and $y = \sqrt{8}\arcsin t$. **COMPUTE** the **arclength** of this parametric curve for $\dfrac{1}{4} \le t \le \dfrac{1}{2}$. **Show** that the answer simplifies to $\boxed{\ln\left(\dfrac{5}{2}\right)}$

11. [18 Points] For each of the following problems, do the following **two** things:

1. Sketch the Polar curves and shade the described bounded region.

2. Set-Up but **DO NOT EVALUATE** the Integral representing the area of the described bounded region.

(a) The **area** bounded outside the polar curve $r = 1+\cos\theta$ and inside the polar curve $r = 3\cos\theta$.

(b) The **area** bounded outside the polar curve $r = 2$ and inside the polar curve $r = 4\sin\theta$.

(c) The **area** that lies inside both of the curves $r = 1+\sin\theta$ and inside the polar curve $r = 1-\sin\theta$.