Math 121 Final Exam Dec 13, 2021

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- Numerical answers $\sin\left(\frac{\pi}{6}\right)$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, $e^{3\ln 3}$, or $\arctan(\sqrt{3})$, should be simplified.
- Please *show* all of your work and *justify* all of your answers.

1. [16 Points] (a) Use Series to show that $\displaystyle\lim_{x\to 0} \frac{\ln(1+x) - x + \frac{x^2}{2}}{2x - \sin(2x)} = \frac{1}{4}$

(b) Compute $\displaystyle\lim_{x\to 0} \frac{\ln(1+x) - x + \frac{x^2}{2}}{2x - \sin(2x)}$ again using L'Hôpital's Rule. Yes! **Three** L'H Rules

2. [20 Points] Compute the following integral. Simplify.

(a) $\displaystyle\int x^4 \arcsin x \, dx$ (b) $\displaystyle\int \frac{1}{(x^2+4)^2}\, dx$

3. [26 Points] For each of the following **Improper** integrals, determine whether it Converges or Diverges. If it converges, find its value. Simplify.

(a) $\displaystyle\int_{-\infty}^{-1} \frac{1}{x^2 - 6x + 25}\, dx$ (b) $\displaystyle\int_{-1}^{6} \frac{15 - x}{x^2 - 6x - 7}\, dx$ (c) $\displaystyle\int_{-1}^{0} \frac{e^{\frac{1}{x}}}{x^2}\, dx$

4. [25 Points] Name any convergence test(s) you use, justify all of your work.

(a) Use the Absolute Convergence Test to show that $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^6 + 2021}$ Converges.

(b) Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{n^6}{\ln(n+2021)}$ Converges or Diverges.

(c) Use the Integral Test to determine whether $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$ Converges or Diverges.

Skip Preconditions. Hint: $\displaystyle\int_2^{\infty} \frac{\ln x}{x^3}\, dx = \int_2^{\infty} \ln x \cdot x^{-3}\, dx$

5. [15 Points] Name any convergence test(s) you use, justify all of your work.

(a) Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ is Absolutely or Conditionally Convergent.

(b) Determine whether $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\ln(2021)}{n^6} + \frac{1}{6^n}\right)$ is Absolutely or Conditionally Convergent.

6. [24 Points] Find the **sum** of each of the following series (which do converge). Simplify.

(a) $-\frac{4}{3}+\frac{4}{5}-\frac{4}{7}+\frac{4}{9}-\ldots$ (b) $-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\ldots$ (c) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\,(2^4)\,\pi^{2n+1}}{(\sqrt{2})^{4n}\,(2n)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n\, 9^n\, \pi^{2n}}{2^{2n}\,(2n+1)!}$ (e) $\sum_{n=0}^{\infty} \frac{(-1)^n\, 2^{n+1}\,(\ln 9)^n}{n!}$ (f) Show $\sum_{n=0}^{\infty} \frac{(-3)^n - 2}{4^n} = -\frac{44}{21}$

7. [20 Points] Find the **Interval** and **Radius** of Convergence for the Series

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n\,(5x+7)^n}{(5n+7)\;\;8^n}$ (b) $\sum_{n=1}^{\infty} \frac{n^n\,(\ln n)\,(x-7)^n}{(2n)!\;e^n}$

8. [15 Points] (a) Use Series to **Estimate** $\int_0^1 x^4 e^{-x^3}\,dx$ with error less than $\frac{1}{20}$. Simplify.

(b) Find the Series Representation for $\sinh x$. **Use** the formula $\sinh x = \frac{e^x - e^{-x}}{2}$

9. [15 Points] (a) Use Series to compute $\int 5x^2 \arctan(5x)\,dx$. **State** the Radius of Convergence.

(b) Find the MacLaurin Series Representation for $\frac{1}{(1+7x)^2}$. **State** the Radius of Convergence.

Hint: $\frac{1}{(1+7x)^2} = \frac{d}{dx}\left(\frac{-1}{7(1+7x)}\right)$

10. [24 Points] For **each** of the following problems, do the following **THREE** things:

1. **Sketch** the Polar curve(s) and **shade** the described bounded region.

2. Set-Up but **DO NOT EVALUATE** an Integral representing the area of the described bounded region.

3. Set-Up but **DO NOT EVALUATE** another **slightly different** Integral representing the same area of the described bounded region.

(a) The **area** bounded outside the polar curve $r = 2$ and inside $r = 4\cos\theta$.

(b) The **area** that lies inside both of the curves $r = 3 + 3\cos\theta$ and $r = 3 - 3\cos\theta$.

(c) The **area** bounded outside the polar curve $r = 1 - \sin\theta$ and inside $r = -3\sin\theta$.

(d) The **area** bounded inside both of the polar curves $r = 2\cos\theta$ and $r = 2\sin\theta$.