Math 121 Final Exam December 19, 2018

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, $e^{3\ln 3}$, $\arctan(\sqrt{3})$, or $\cosh(\ln 3)$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [18 Points] Evaluate each of the following **limits**. Please justify your answer. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist. Simplify.

(a) $\displaystyle\lim_{x\to 0} \frac{xe^x - \sin x}{\ln(1+x) - \arctan x}$ (b) Compute $\displaystyle\lim_{x\to 0} \frac{xe^x - \sin x}{\ln(1+x) - \arctan x}$ **again** using series.

(c) $\displaystyle\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^x$

2. [20 Points] Evaluate the following **integral**.

(a) Show that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{\left(1+\sin^2 x\right)^{\frac{7}{2}}}\,dx = \frac{43}{60\sqrt{2}}$ (b) $\displaystyle\int_1^{\sqrt{3}} \frac{x^2}{\sqrt{4-x^2}}\,dx$

3. [40 Points] For each of the following **improper integrals**, determine whether it converges or diverges. If it converges, find its value. Simplify.

(a) $\displaystyle\int_0^5 \frac{6}{x^2-4x-5}\,dx$ (b) $\displaystyle\int_6^\infty \frac{6}{x^2-4x-5}\,dx$ Tip: Reuse your algebra work from part (a).

(c) $\displaystyle\int_{-\infty}^5 \frac{6}{x^2-4x+7}\,dx$ (d) $\displaystyle\int_1^2 \frac{1}{x\ln x}\,dx$ (e) $\displaystyle\int_0^e \frac{\ln x}{\sqrt{x}}\,dx$

4. [18 Points] Find the **sum** of each of the following series (which do converge). Simplify.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 5^{2n+1}}{2^{5n-1}}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\,(\ln 9)^n}{2^{n+1}\cdot n!}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\pi^{2n-1}}{9^n\,(2n)!}$

(d) $-\dfrac{\pi^3}{3!} + \dfrac{\pi^5}{5!} - \dfrac{\pi^7}{7!} + \dfrac{\pi^9}{9!} - \ldots$ (e) $-1 + \dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{4} - \dfrac{1}{5} + \ldots$ (f) $4 - \dfrac{4}{3} + \dfrac{4}{5} - \dfrac{4}{7} + \dfrac{4}{9} - \ldots$

5. [30 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **divergent**. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2+7}{n^7+2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(7n)}{7^n} + \frac{7}{n^7}$ (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+7}{n^2}\right)$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+7)^7}{\ln(n+7)}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,(3n)!\,\ln n}{(n!)^2\,e^{4n}\,n^n}$

6. [20 Points] Find the **Interval** and **Radius** of Convergence for the following power series. Analyze carefully and with full justification.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (3x-5)^n}{(n+7)^2 \cdot 7^{n+1}}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ x^{2n}}{(2n)!}$ (c) $\displaystyle\sum_{n=1}^{\infty} n!\ (x-6)^n$

7. [12 Points] Please analyze with detail and justify carefully. Simplify.

(a) Use MacLaurin Series to **Estimate** $\displaystyle\int_0^1 x^3 \ln\left(1+x^3\right)\ dx$ with error less than $\dfrac{1}{30}$.

(b) Use MacLaurin Series to **Estimate** $e^{-\frac{1}{3}}$ with error less than $\dfrac{1}{100}$.

8. [8 Points] For both parts, you do **not** need to find the Radius of Convergence

(a) Demonstrate one method to compute the MacLaurin Series for $f(x) = \sin x$. Justify. Do not just write down the formula.

(b) Demonstrate a second, **different** method to compute the MacLaurin Series for $f(x) = \sin x$. Justify. Do not just write down a formula.

9. [16 Points]

(a) Consider the region bounded by $y = e^x + 2$, $y = \sin x$, $x = 0$ and $x = \pi$. Rotate the region about the horizontal line $\boxed{\boldsymbol{y = -1}}$. **Set-Up** but **DO NOT EVALUATE** the integral representing the **volume** of the resulting solid using the Washer Method. Sketch the solid, along with one of the approximating washer.

(b) Consider the same region bounded by $y = e^x + 2$, $y = \sin x$, $x = 0$ and $x = \pi$. Rotate the region about the vertical line $\boxed{\boldsymbol{x = -2}}$. **Set-Up** but **DO NOT EVALUATE** the integral representing the **volume** of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

(c) Consider the region bounded by $y = \arcsin x$, $y = \dfrac{\pi}{2}$, $x = 0$ and $x = 1$. Rotate the region about the vertical line $\boxed{\boldsymbol{x = 3}}$. **Set-Up** but **DO NOT EVALUATE** the integral representing the **volume** of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

10. [18 Points]

(a) Consider the Parametric Curve represented by $x = \ln t + \ln(1-t^2)$ and $y = \sqrt{8} \arcsin t$.

COMPUTE the **arclength** of this parametric curve for $\dfrac{1}{4} \le t \le \dfrac{1}{2}$. **Show** that the answer simplifies to $\boxed{\ln\left(\dfrac{5}{2}\right)}$

(b) Consider a different Parametric Curve represented by $x = t - e^{2t}$ and $y = 1 - \sqrt{8} e^t$.

COMPUTE the **surface area** obtained by rotating this curve about the y-axis for $0 \le t \le 3$. **Show** that the answer simplifies to $\boxed{2\pi\left(6 + 2e^6 - \dfrac{e^{12}}{2}\right)}$

NOTE: NO Polar on Fall 2018 Final Exam.