Math 121 Final Exam December 22, 2017

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, $e^{3\ln 3}$, $\arctan(\sqrt{3})$, or $\cosh(\ln 3)$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [20 Points] Evaluate the following **limits**. Please justify your answer. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist. Simplify.

(a) $\displaystyle\lim_{x\to 0} \frac{(\sin(3x)) - 3x}{x - \arctan x}$ (b) Compute $\displaystyle\lim_{x\to 0} \frac{(\sin(3x)) - 3x}{x - \arctan x}$ **again** using series.

(c) $\displaystyle\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^x$

2. [10 Points] Evaluate the following **integral**. $\displaystyle\int \frac{\cos x}{(4+\sin^2 x)^{\frac{5}{2}}}\,dx$

3. [40 Points] For the following **improper integral**, determine whether it converges or diverges. If it converges, find its value. Simplify.

(a) $\displaystyle\int_0^1 \frac{x^3+4x+3}{x^3+3x}\,dx = \int_0^1 \frac{x^3+4x+3}{x(x^2+3)}\,dx$ (b) $\displaystyle\int_{-\infty}^5 \frac{6}{x^2-4x+7}\,dx$

(c) $\displaystyle\int_6^\infty \frac{6}{x^2-4x-5}\,dx$ (d) $\displaystyle\int_0^e \frac{\ln x}{\sqrt{x}}\,dx$

4. [18 Points] Find the **sum** of each of the following series (which do converge). Simplify.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 5^{n+1}}{2^{3n-1}}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\, 2^{n+1}\, (\ln 9)^n}{n!}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{9^{n-1}\,(2n+1)!}$

(d) $\frac{1}{3}-\frac{1}{5}+\frac{1}{7}-\frac{1}{9}+\ldots$ (e) $1-\frac{\pi^2}{(4)2!}+\frac{\pi^4}{(16)4!}-\frac{\pi^6}{(64)6!}+\frac{\pi^8}{(256)8!}-\ldots$ (f) $-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\ldots$

5. [30 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **divergent**. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,(\sqrt{n}+5)}{n^2+2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+4)^2}{\ln(n+4)}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+1}{n^2}\right)$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!\,\ln n}{(n!)^2\, e^{4n}\, n^n}$

6. [15 Points] Find the **Interval** and **Radius** of Convergence for the following power series. Analyze carefully and with full justification.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \, (5x+1)^n}{(n+7)^2 \cdot 9^n}$$

7. [12 Points] Please analyze with detail and justify carefully. Simplify.

(a) Use MacLaurin series to **Estimate** $\displaystyle\int_0^1 x^2 e^{-x^3} \, dx$ with error less than $\dfrac{1}{50}$.

(b) Use MacLaurin Series to **Estimate** $\sin(1)$ with error less than $\dfrac{1}{1000}$.

Tip: $7! = 5040$

8. [10 Points] For the following, you do not need to compute the Radius of Convergence here.

(a) Compute the MacLaurin Series for $\boldsymbol{\cosh x}$, any way that you know how to. Justify all details.

(b) Demonstrate a **second**, **different** method/approach from part (a) above, to compute the MacLaurin Series for the same function, $\boldsymbol{\cosh x}$.

OPTIONAL BONUS (c) Demonstrate a **third**, **different** method/approach from parts (a) and (b) above, to compute the MacLaurin Series for the same function, $\boldsymbol{\cosh x}$.

9. [15 Points] Consider the region bounded by $y = \arcsin x$, $y = 0$, $x = 0$ and $x = 1$. Rotate the region about the $\boxed{y\text{-axis}}$. **COMPUTE** the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

10. [15 Points] Consider the Parametric Curve represented by $x = 3 - 2t$ and $y = e^t + e^{-t}$. **COMPUTE** the **Surface Area** obtained by rotating this curve about the $\boxed{y\text{-axis}}$ for $0 \le t \le 1$. Simplify.

11. [15 Points] **COMPUTE** the area bounded outside the polar curve $r = 1 + \sin\theta$ and inside the polar curve $r = 3\sin\theta$. Sketch the Polar curves and shade the described bounded region.