• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

•You need *NOT* simplify algebraically complicated answers. However, numerical answers such as $\sin\frac{\pi}{6}$, $\arctan\left(\sqrt{3}\right)$, $4^{3/2}$, $e^{\ln 4}$, $\ln e^7$, $e^{-\ln 5}$, $e^{3\ln 3}$, or $\cosh(\ln 3)$ should be simplified.

•Please read each question carefully. *Show all of your work* and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$, $-\infty$, or Does Not Exist. [15]

   (a) $\displaystyle\lim_{x\to 0} \frac{\sinh x - x}{\arctan(4x) - e^{4x} + 1}$ (b) $\displaystyle\lim_{x\to\infty} x^{\frac{\ln 3}{2+\ln x}}$

2. Evaluate each of the following **integrals**. [30]

   (a) $\displaystyle\int_2^{2\sqrt{3}} \frac{1}{\sqrt{16-x^2}} + \frac{1}{4+x^2}\,dx$ (b) $\displaystyle\int \frac{1}{(x^2+4)^{\frac{5}{2}}}\,dx$ (c) $\displaystyle\int \frac{x^4+2x^2+x+2}{x^3+2x}\,dx$

3. For each of the following **improper integrals**, determine whether it converges or diverges. If it converges, find its value. [20]

   (a) $\displaystyle\int_9^{\infty} \frac{1}{x^2-8x+41}\,dx$ (b) $\displaystyle\int_0^9 \frac{1}{(x-1)^{\frac{5}{3}}}\,dx$

4. Find the **sum** of each of the following series (which do converge): [15]

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 3^{n-1}}{2^{2n+1}}$ (b) $\displaystyle\frac{\pi}{6} - \frac{1}{3!}\left(\frac{\pi}{6}\right)^3 + \frac{1}{5!}\left(\frac{\pi}{6}\right)^5 - \frac{1}{7!}\left(\frac{\pi}{6}\right)^7 + \frac{1}{9!}\left(\frac{\pi}{6}\right)^9 - \cdots$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+1}(\ln 3)^n}{n!}$

5. In each case determine whether the given series is **absolutely convergent, conditionally convergent**, or **divergent**. Justify your answers. [25]

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\left(3n^5+6\right)}{n^9+n-1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,(3n)!\;n^n}{(n!)^4\,2^{4n}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{9n-2}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{7+n^2}{5n^2-n+14}$

6. Find the **Interval** and **Radius** of Convergence for the following power series: $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(3x-1)^n}{4^n \sqrt{n}}$ [15]

   Analyze carefully and with justification.

7. Please analyze with detail and justify carefully. [10]

   (a) Find the **MacLaurin series** representation for $f(x) = x \ln\left(1 + x^3\right)$.

   (b) Use this series to determine the **sixth** and **seventh** derivatives of $f(x) = x \ln\left(1 + x^3\right)$ at $x = 0$. (You do **NOT** need to simplify your answers.)

8. Please analyze with detail and justify carefully. [15]

   (a) Find the **MacLaurin series** representation for $f(x) = x^3 \cos\left(x^2\right)$.

   Your answer should be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$.

   (b) Use the MacLaurin series representation for $f(x) = x^3 \cos\left(x^2\right)$ from part (a) to

   $$\textbf{Estimate} \int_0^1 x^3 \cos\left(x^2\right) dx \text{ with error less than } \frac{1}{100}.$$

9. Volumes of Revolution [20]

   (a) Consider the region bounded by $y = e^x$, and $y = \ln x$, between $x = 1$ and $x = 2$. Rotate the region about the y–axis. **Compute** the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.

   (b) Consider the region bounded by $y = \arctan x$, and $y = 0$, between $x = 0$ and $x = 1$. Rotate the region about the line $x = 6$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating cylindrical shells.

10. Parametric Curves [20]

    (a) Consider the Parametric Curve represented by $x = \dfrac{t^3}{3} - \dfrac{e^{2t}}{2}$ and $y = 2te^t - 2e^t$.
    Compute the **arclength** of this parametric curve for $0 \le t \le 1$.

    (b) Consider a different Parametric Curve represented by $x = e^t \cos t$ and $y = e^t \sin t$. Set-up, **BUT DO NOT EVALUATE!!**, the definite integral representing the **surface area** obtained by rotating this curve about the y-axis, for $0 \le t \le \ln \pi$.

11. Compute the **area** bounded outside the polar curve $r = 1 + \cos\theta$ and inside the polar curve $r = 3\cos\theta$. **Sketch** the Polar curves. [15]