

$$\begin{aligned}
 1. \text{ Compute } \int_e^{e^3} \frac{4}{x(\ln x)^2} dx &= 4 \int_1^3 \frac{1}{u^2} du = 4 \int_1^3 u^{-2} du = -4u^{-1} \Big|_1^3 = -\frac{4}{u} \Big|_1^3 = -\frac{4}{3} - (-4) \\
 &= -\frac{4}{3} + 4 = -\frac{4}{3} + \frac{12}{3} = \boxed{\frac{8}{3}}
 \end{aligned}$$

Here $\begin{cases} u = \ln x \\ du = \frac{1}{x} dx \end{cases}$ and $\begin{cases} x = e \implies u = \ln e = 1 \\ x = e^3 \implies u = \ln e^3 = 3 \end{cases}$

$$\begin{aligned}
 2. \text{ Compute } \int_{\ln 3}^{\ln 8} \frac{e^x}{\sqrt{1+e^x}} dx &= \int_4^9 \frac{1}{\sqrt{u}} du = \int_4^9 u^{-\frac{1}{2}} du = 2\sqrt{u} \Big|_4^9 \\
 &= 2\sqrt{9} - 2\sqrt{4} = 2(3) - 2(2) = 6 - 4 = \boxed{2}
 \end{aligned}$$

Here $\begin{cases} u = 1 + e^x \\ du = e^x dx \end{cases}$ and $\begin{cases} x = \ln 3 \implies u = 1 + e^{\ln 3} = 1 + 3 = 4 \\ x = \ln 8 \implies u = 1 + e^{\ln 8} = 1 + 8 = 9 \end{cases}$

$$\begin{aligned}
 3. \text{ Compute } \int_{\ln 2}^{\ln 3} \frac{1}{e^{2x}(1-e^{-2x})^2} dx &= \frac{1}{2} \int_{\frac{3}{4}}^{\frac{8}{9}} \frac{1}{u^2} du = -\frac{1}{2u} \Big|_{\frac{3}{4}}^{\frac{8}{9}} = -\frac{1}{2} \left(\frac{1}{(\frac{8}{9})} - \frac{1}{(\frac{3}{4})} \right) \\
 &= -\frac{1}{2} \left(\frac{9}{8} - \frac{4}{3} \right) = -\frac{1}{2} \left(\frac{27}{24} - \frac{32}{24} \right) = -\frac{1}{2} \left(-\frac{5}{24} \right) = \boxed{\frac{5}{48}}
 \end{aligned}$$

Here $\begin{cases} u = 1 - e^{-2x} \\ du = 2e^{-2x} dx \\ \frac{1}{2} du = \frac{1}{e^{2x}} dx \end{cases}$ and $\begin{cases} x = \ln 2 \implies u = 1 - e^{-2\ln 2} = 1 - e^{\ln(2^{-2})} = 1 - \frac{1}{4} = \frac{3}{4} \\ x = \ln 3 \implies u = 1 - e^{-2\ln 3} = 1 - e^{\ln(3^{-2})} = 1 - \frac{1}{9} = \frac{8}{9} \end{cases}$

$$\begin{aligned}
 4. \text{ Compute } \int \frac{x}{(3x+1)^2} dx &= \frac{1}{3} \int \frac{\left(\frac{u-1}{3}\right)}{u^2} du = \frac{1}{9} \int \frac{u-1}{u^2} du \\
 &= \frac{1}{9} \int \frac{u}{u^2} - \frac{1}{u^2} du = \frac{1}{9} \int \frac{1}{u} - u^{-2} du \\
 &= \frac{1}{9} \left(\ln|u| + \frac{1}{u} \right) + C = \boxed{\frac{1}{9} \left(\ln|3x+1| + \frac{1}{3x+1} \right) + C}
 \end{aligned}$$

Here

$$\begin{aligned} u &= 3x + 1 \Rightarrow x = \frac{u - 1}{3} \\ du &= 3dx \\ \frac{1}{3}du &= dx \end{aligned}$$

5. Compute $\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \tan(3x) dx = \int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \frac{\sin(3x)}{\cos(3x)} dx = -\frac{1}{3} \int_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} \frac{1}{u} du = -\frac{1}{3} \ln |u| \Big|_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}}$

$$= -\frac{1}{3} \left(\ln \left(\frac{1}{2} \right) - \ln \left(\frac{\sqrt{3}}{2} \right) \right) = -\frac{1}{3} \left(\ln \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) \right)$$

$$= -\frac{1}{3} \left(\ln \left(\frac{1}{\sqrt{3}} \right) \right) = -\frac{1}{3} (\ln 1 - \ln \sqrt{3}) = -\frac{1}{3} (0 - \ln \sqrt{3}) = \boxed{\frac{\ln \sqrt{3}}{3}} \text{ or } \boxed{\frac{\ln 3}{6}}$$

Here

$$\begin{aligned} u &= \cos(3x) \\ du &= -3 \sin(3x) dx \\ -\frac{1}{3} du &= \sin(3x) dx \end{aligned}$$

and

$$\begin{aligned} x = \frac{\pi}{18} &\implies u = \cos\left(\frac{3\pi}{18}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \\ x = \frac{\pi}{9} &\implies u = \cos\left(\frac{3\pi}{9}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \end{aligned}$$

6. Consider $G(x) = \frac{1}{\sin \sqrt{e^x + e^7}} + \frac{1}{e^{\sqrt{x^2 + 7 \sin x}}} + \frac{1}{\sqrt{7 + e^{\sin x}}}$ Compute $G'(x)$. Do not simplify here.

First rewrite to simplify and prepare:

$$G(x) = (\sin \sqrt{e^x + e^7})^{-1} + e^{-\sqrt{x^2 + 7 \sin x}} + (7 + e^{\sin x})^{-\frac{1}{2}}$$

$$G'(x) = \boxed{- (\sin \sqrt{e^x + e^7})^{-2} \cos \sqrt{e^x + e^7} \left(\frac{1}{2\sqrt{e^x + e^7}} \right) (e^x)} \text{ (continued ...)}$$

$$\boxed{+ e^{-\sqrt{x^2 + 7 \sin x}} \left(-\frac{1}{2\sqrt{x^2 + 7 \sin x}} \right) (2x + 7 \cos x)} \text{ (continued ...)}$$

$$\boxed{-\frac{1}{2} (7 + e^{\sin x})^{-\frac{3}{2}} (e^{\sin x}) (\cos x)}$$

7. Consider $F(x) = \sin(\ln(1+x)) - \frac{1}{1 + \ln(1+3x)}$ Compute the equation of the tangent line to the curve $F(x)$ at the point where $x = 0$.

First compute the corresponding y -value.

$$F(0) = \sin(\ln(1+0)) - \frac{1}{1+\ln(1+0)} = \sin(0) - \frac{1}{1+0} = 0 - 1 = -1$$

Second, compute the derivative for $F(x) = \sin(\ln(1+x)) - (1+\ln(1+3x))^{-1}$

$$F'(x) = \cos(\ln(1+x)) \left(\frac{1}{1+x} \right) + \frac{1}{(1+\ln(1+3x))^2} \left(\frac{1}{1+3x} \right) \quad (3)$$

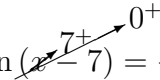
Next find the specific slope when $x = 0$.

$$\begin{aligned} F'(0) &= \cos(\ln(1+0)) \left(\frac{1}{1+0} \right) + \frac{1}{(1+\ln(1+0))^2} \left(\frac{1}{1+0} \right) \quad (3) \\ &= \cos(0)(1) + \frac{1}{(1+0)^2}(1)(3) = (1)(1) + (1)(1)(3) = 4 \end{aligned}$$

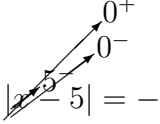
Finally, use *point-slope form* to get the equation of the tangent line $y - (-1) = 4(x - 0)$.

We have $\boxed{y = 4x - 1}$

8. $\lim_{x \rightarrow 7^+} \ln(x-7) = \lim_{x \rightarrow 7^+} \ln(x-7) = -\infty$ The arrows help justify the size argument(s).



9. $\lim_{x \rightarrow 5^-} \ln|x-5| = \lim_{x \rightarrow 5^-} \ln|x-5| = -\infty$



10. $\lim_{x \rightarrow -6^-} \ln|x+6| = \lim_{x \rightarrow -6^-} \ln|x+6| = -\infty$

