1. Compute $\displaystyle\int_e^{e^3} \frac{4}{x(\ln x)^2}\, dx = 4\int_1^3 \frac{1}{u^2}\, du = 4\int_1^3 u^{-2}\, du = -4u^{-1}\Big|_1^3 = -\frac{4}{u}\Big|_1^3 = -\frac{4}{3} - (-4)$

$= -\dfrac{4}{3} + 4 = -\dfrac{4}{3} + \dfrac{12}{3} = \boxed{\dfrac{8}{3}}$

Here $\boxed{\begin{aligned} u &= \ln x \\ du &= \tfrac{1}{x}\, dx \end{aligned}}$ and $\boxed{\begin{aligned} x = e &\implies u = \ln e = 1 \\ x = e^3 &\implies u = \ln e^3 = 3 \end{aligned}}$

2. Compute $\displaystyle\int_{\ln 3}^{\ln 8} \frac{e^x}{\sqrt{1+e^x}}\, dx = \int_4^9 \frac{1}{\sqrt{u}}\, du = \int_4^9 u^{-\frac{1}{2}}\, du = 2\sqrt{u}\Big|_4^9$

$= 2\sqrt{9} - 2\sqrt{4} = 2(3) - 2(2) = 6 - 4 = \boxed{2}$

Here $\boxed{\begin{aligned} u &= 1 + e^x \\ du &= e^x\, dx \end{aligned}}$ and $\boxed{\begin{aligned} x = \ln 3 &\implies u = 1 + e^{\ln 3} = 1 + 3 = 4 \\ x = \ln 8 &\implies u = 1 + e^{\ln 8} = 1 + 8 = 9 \end{aligned}}$

3. Compute $\displaystyle\int_{\ln 2}^{\ln 3} \frac{1}{e^{2x}\left(1 - e^{-2x}\right)^2}\, dx = \frac{1}{2}\int_{\frac{3}{4}}^{\frac{8}{9}} \frac{1}{u^2}\, du = -\frac{1}{2u}\Big|_{\frac{3}{4}}^{\frac{8}{9}} = -\frac{1}{2}\left(\frac{1}{\left(\frac{8}{9}\right)} - \frac{1}{\left(\frac{3}{4}\right)}\right)$

$= -\dfrac{1}{2}\left(\dfrac{9}{8} - \dfrac{4}{3}\right) = -\dfrac{1}{2}\left(\dfrac{27}{24} - \dfrac{32}{24}\right) = -\dfrac{1}{2}\left(-\dfrac{5}{24}\right) = \boxed{\dfrac{5}{48}}$

Here $\boxed{\begin{aligned} u &= 1 - e^{-2x} \\ du &= 2e^{-2x} dx \\ \frac{1}{2} du &= \frac{1}{e^{2x}} dx \end{aligned}}$ and $\boxed{\begin{aligned} x = \ln 2 &\implies u = 1 - e^{-2\ln 2} = 1 - e^{\ln\left(2^{-2}\right)} = 1 - \frac{1}{4} = \frac{3}{4} \\ x = \ln 3 &\implies u = 1 - e^{-2\ln 3} = 1 - e^{\ln\left(3^{-2}\right)} = 1 - \frac{1}{9} = \frac{8}{9} \end{aligned}}$

4. Compute $\displaystyle\int \frac{x}{(3x+1)^2}\, dx = \frac{1}{3}\int \frac{\left(\frac{u-1}{3}\right)}{u^2}\, du = \frac{1}{9}\int \frac{u-1}{u^2}\, du$

$= \dfrac{1}{9}\displaystyle\int \frac{u}{u^2} - \frac{1}{u^2}\, du = \frac{1}{9}\int \frac{1}{u} - u^{-2}\, du$

$= \dfrac{1}{9}\left(\ln|u| + \dfrac{1}{u}\right) + C = \boxed{\dfrac{1}{9}\left(\ln|3x+1| + \dfrac{1}{3x+1}\right) + C}$

Here $\boxed{\begin{aligned} u &= 3x+1 \Rightarrow x = \frac{u-1}{3} \\ du &= 3dx \\ \frac{1}{3}du &= dx \end{aligned}}$

5. Compute $\displaystyle\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \tan(3x)\ dx = \int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \frac{\sin(3x)}{\cos(3x)}\ dx = -\frac{1}{3}\int_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} \frac{1}{u}\ du = -\frac{1}{3}\ln|u|\Big|_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}}$

$$= -\frac{1}{3}\left(\ln\left(\frac{1}{2}\right) - \ln\left(\frac{\sqrt{3}}{2}\right)\right) = -\frac{1}{3}\left(\ln\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\right)$$

$$= -\frac{1}{3}\left(\ln\left(\frac{1}{\sqrt{3}}\right)\right) = -\frac{1}{3}\left(\ln 1 - \ln\sqrt{3}\right) = -\frac{1}{3}\left(0 - \ln\sqrt{3}\right) = \boxed{\frac{\ln\sqrt{3}}{3}} \overset{\text{or}}{=} \boxed{\frac{\ln 3}{6}}$$

Here $\boxed{\begin{aligned} u &= \cos(3x) \\ du &= -3\sin(3x)dx \\ -\tfrac{1}{3}du &= \sin(3x)dx \end{aligned}}$ and $\boxed{\begin{aligned} x = \tfrac{\pi}{18} &\implies u = \cos\left(\tfrac{3\pi}{18}\right) = \cos\left(\tfrac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \\ x = \tfrac{\pi}{9} &\implies u = \cos\left(\tfrac{3\pi}{9}\right) = \cos\left(\tfrac{\pi}{3}\right) = \frac{1}{2} \end{aligned}}$

6. Consider $G(x) = \dfrac{1}{\sin\sqrt{e^x + e^7}} + \dfrac{1}{e^{\sqrt{x^2+7\sin x}}} + \dfrac{1}{\sqrt{7+e^{\sin x}}}$ Compute $G'(x)$. Do not simplify here.

First rewrite to simplify and prepare:

$G(x) = \left(\sin\sqrt{e^x+e^7}\right)^{-1} + e^{-\sqrt{x^2+7\sin x}} + \left(7+e^{\sin x}\right)^{-\frac{1}{2}}$

$G'(x) = \boxed{-\left(\sin\sqrt{e^x+e^7}\right)^{-2}\cos\sqrt{e^x+e^7}\left(\dfrac{1}{2\sqrt{e^x+e^7}}\right)(e^x)}$ (continued ...)

$\boxed{+e^{-\sqrt{x^2+7\sin x}}\left(-\dfrac{1}{2\sqrt{x^2+7\sin x}}\right)(2x+7\cos x)}$ (continued ...)

$\boxed{-\dfrac{1}{2}\left(7+e^{\sin x}\right)^{-\frac{3}{2}}\left(e^{\sin x}\right)(\cos x)}$

7. Consider $F(x) = \sin(\ln(1+x)) - \dfrac{1}{1+\ln(1+3x)}$ Compute the equation of the tangent line to the curve $F(x)$ at the point where $x = 0$.

First compute the corresponding y-value.

$F(0) = \sin(\ln(1+0)) - \dfrac{1}{1+\ln(1+0)} = \sin(0) - \dfrac{1}{1+0} = 0 - 1 = -1$

Second, compute the derivative for $F(x) = \sin(\ln(1+x)) - (1+\ln(1+3x))^{-1}$

$F'(x) = \cos(\ln(1+x))\left(\dfrac{1}{1+x}\right) + \dfrac{1}{(1+\ln(1+3x))^2}\left(\dfrac{1}{1+3x}\right)(3)$

Next find the specific slope when $x = 0$.

$F'(0) = \cos(\ln(1+0))\left(\dfrac{1}{1+0}\right) + \dfrac{1}{(1+\ln(1+0))^2}\left(\dfrac{1}{1+0}\right)(3)$

$= \cos(0)(1) + \dfrac{1}{(1+0)^2}(1)(3) = (1)(1) + (1)(1)(3) = 4$

Finally, use *point-slope form* to get the equation of the tangent line $y - (-1) = 4(x - 0)$. We have $\boxed{y = 4x - 1}$

8. $\lim\limits_{x\to 7^+} \ln(x-7) = \lim\limits_{x\to 7^+} \ln(\underbrace{\underset{7^+}{x} - 7}_{\to 0^+}) = -\infty$ The arrows help justify the size argument(s).

9. $\lim\limits_{x\to 5^-} \ln|x-5| = \lim\limits_{x\to 5^-} \ln|\underbrace{\underset{5^-}{x} - 5}_{\to 0^-}|_{\to 0^+} = -\infty$

10. $\lim\limits_{x\to -6^-} \ln|x+6| = \lim\limits_{x\to -6^-} \ln|\underbrace{\underset{-6^-}{x} + 6}_{\to 0^-}|_{\to 0^+} = -\infty$