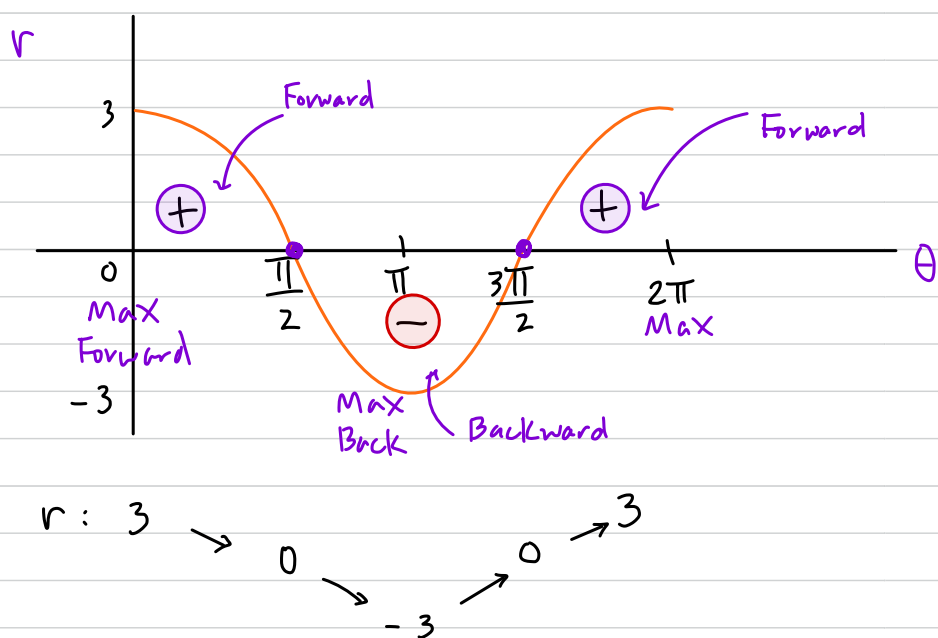# Worksheet 12 Answer Key

1. $r = 3\cos\theta$

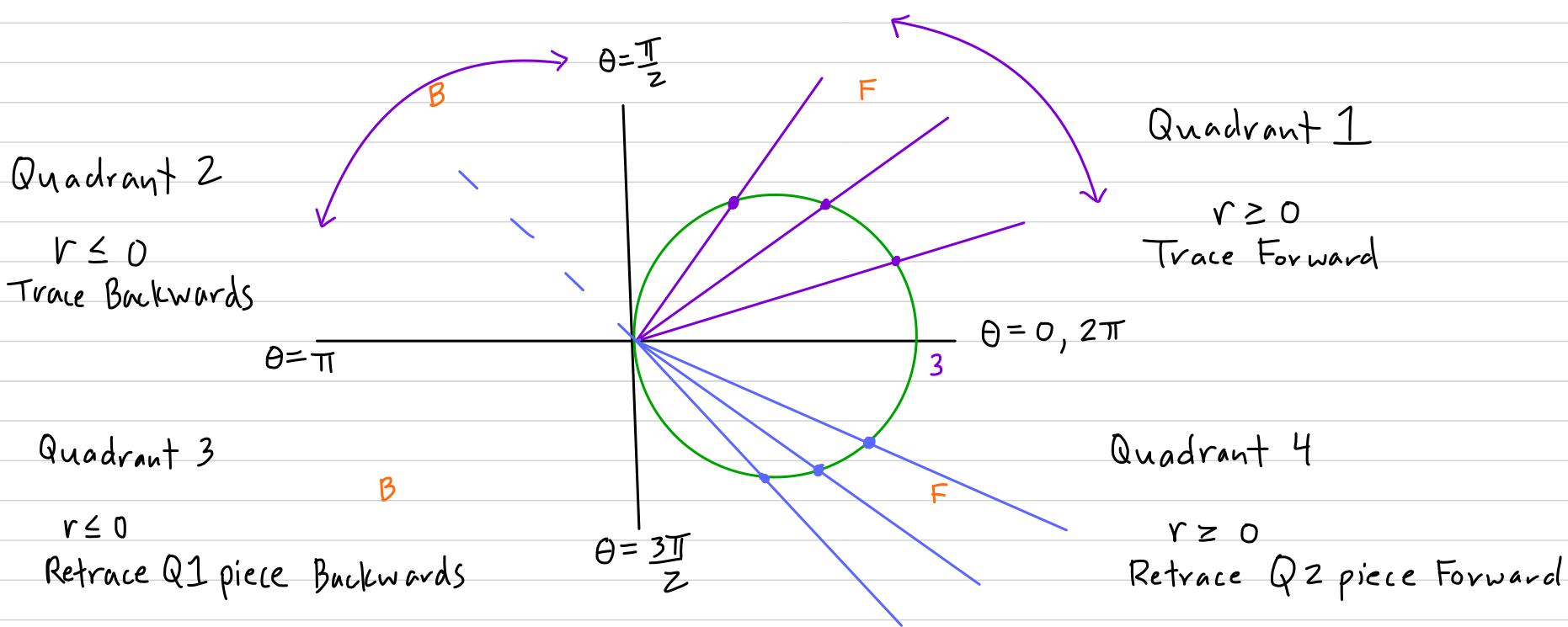Cartesian Plot

$r = 3\cos\theta$



r: $3 \searrow 0 \searrow -3 \nearrow 0 \nearrow 3$
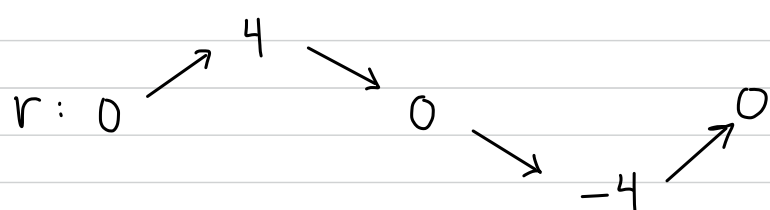
Polar Plot                     $r = 3\cos\theta$



Quadrant 2

$r \leq 0$
Trace Backwards

Quadrant 3

$r \leq 0$
Retrace Q1 piece Backwards

$\theta = \pi$

$\theta = \frac{\pi}{2}$

$\theta = \frac{3\pi}{2}$

$\theta = 0, 2\pi$

Quadrant 1

$r \geq 0$
Trace Forward

Quadrant 4

$r \geq 0$
Retrace Q2 piece Forward
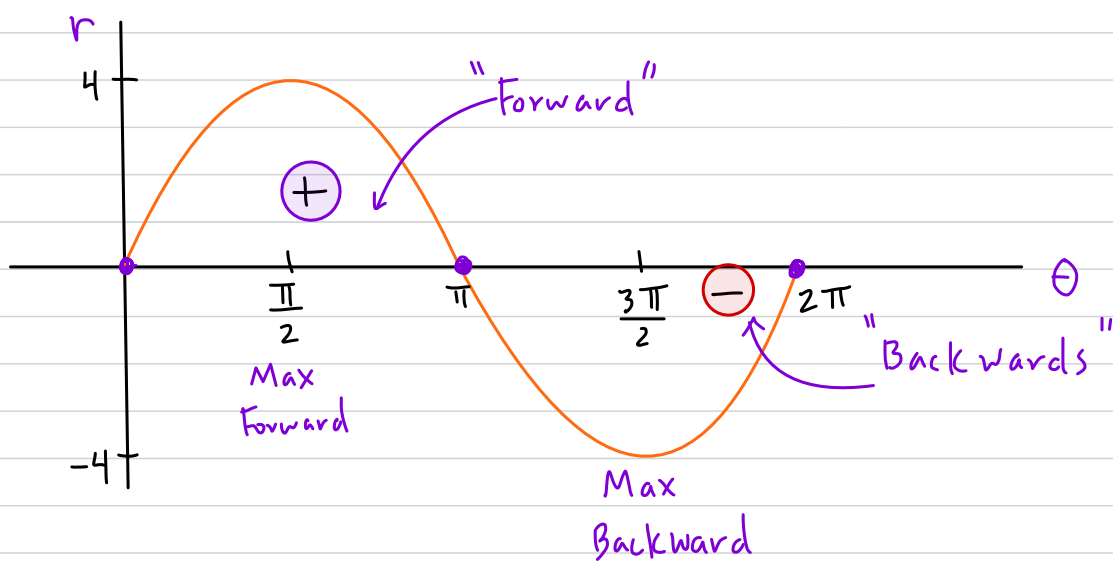
Note: Circle closes one Full Loop as θ ranges from 0 to π.

Then it retraces the Circle as θ ranges from π to 2π.

2. $r = 4\sin\theta$

## Cartesian Plot

$r = 4\sin\theta$

r

4

"Forward"

$+$

Max
Forward

$\frac{\pi}{2}$   $\pi$   $\frac{3\pi}{2}$   $-$   $2\pi$   $\theta$

$-4$

"Backwards"

Max
Backward

$r: 0 \nearrow 4 \searrow 0 \searrow -4 \nearrow 0$

## Polar Plot

$\theta = \frac{\pi}{2}$

$r = 4\sin\theta$

Q2: $r \geq 0$
   Trace Forwards

4

Q1: $r \geq 0$
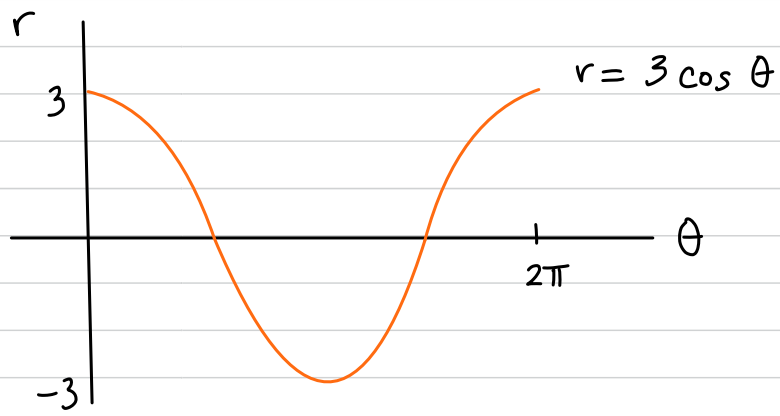   Trace Forwards

$\theta = \pi$

$\theta = 0, 2\pi$

Q3: $r \leq 0$
   ReTrace Backwards

Q4: $r \leq 0$
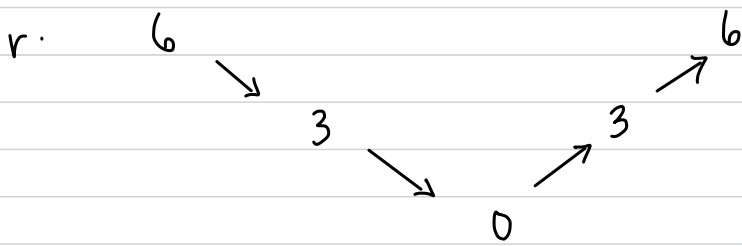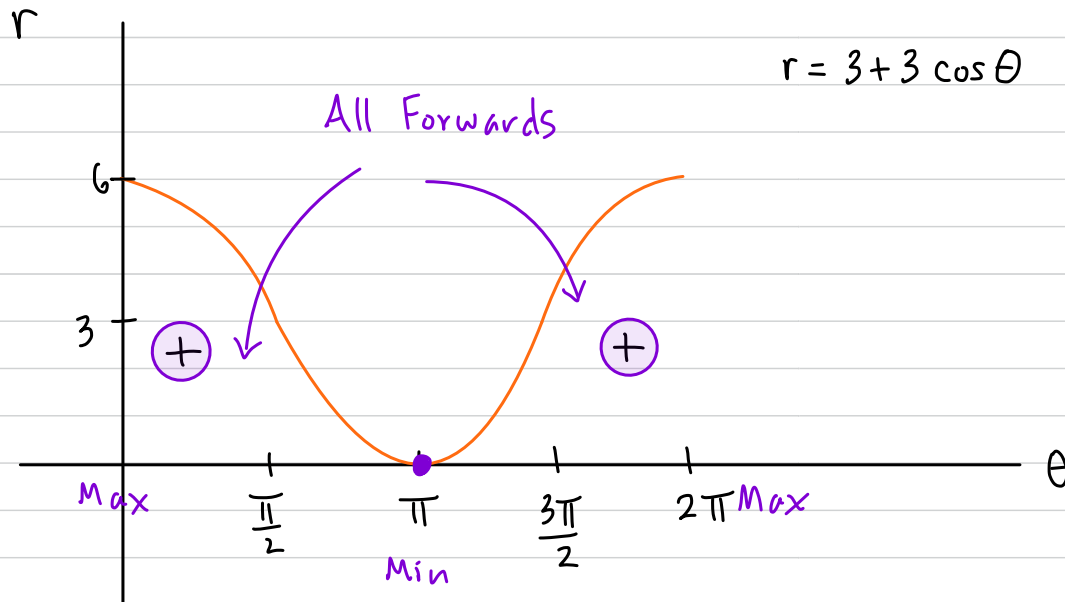   ReTrace Backwards

$\theta = \frac{3\pi}{2}$
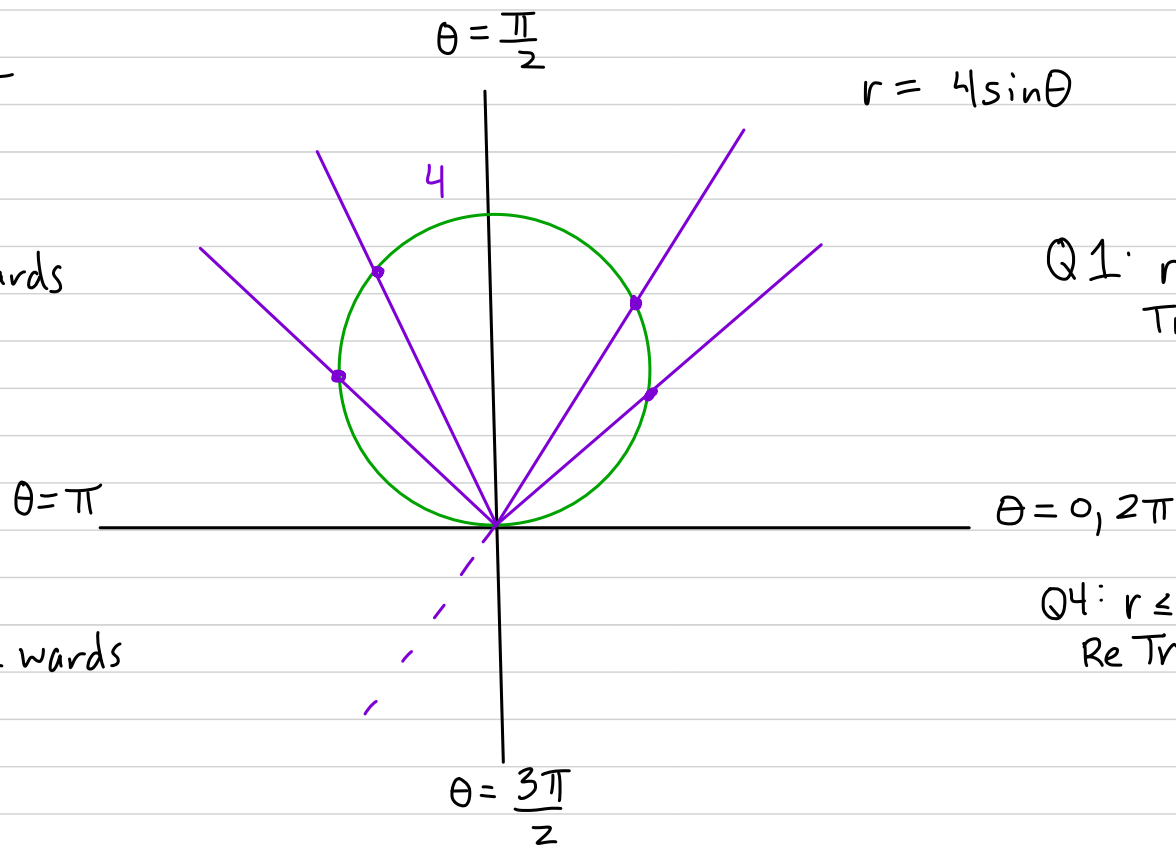
Note: Circle closes one full loop as $\theta$ ranges from 0 to $\pi$ "Forwards"
   Then it retraces the circle as $\theta$ ranges from $\pi$ to $2\pi$ "Backwards"
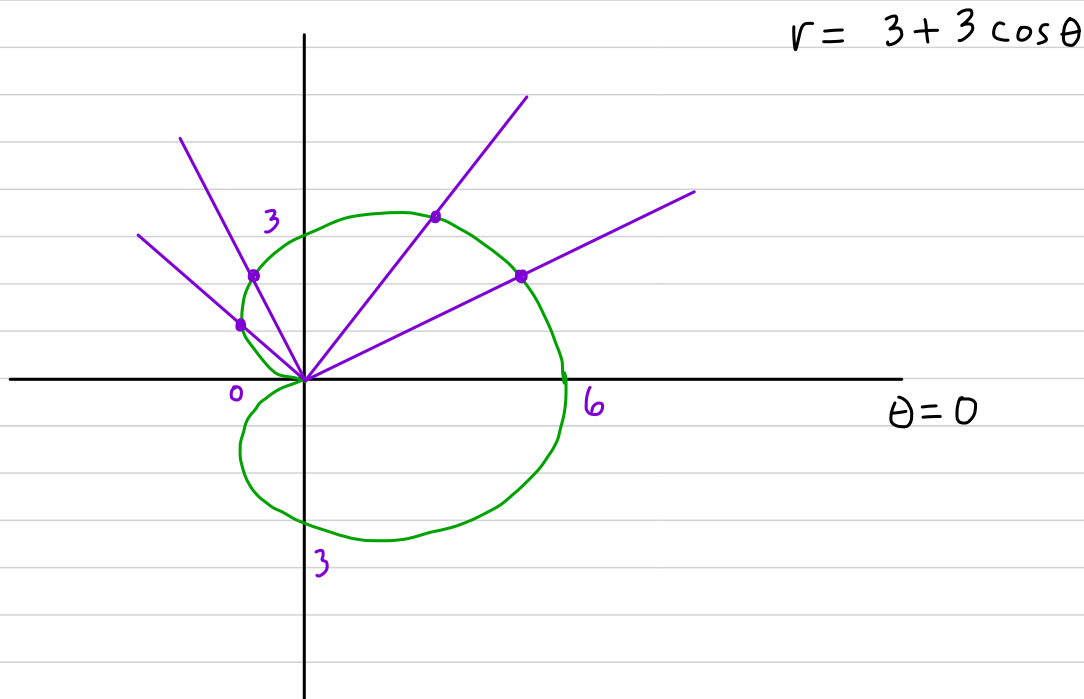   Repeat

3  $r = 3 + 3\cos\theta$

$r = \cos\theta$



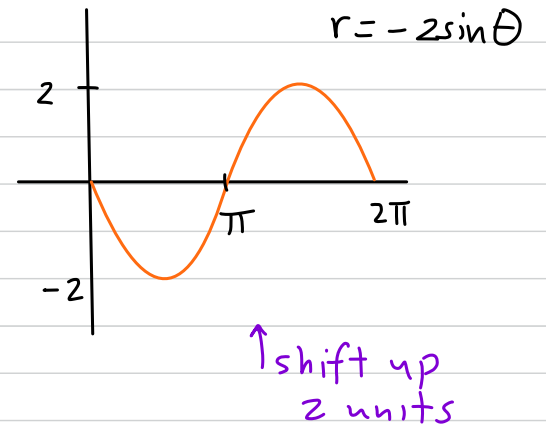$r = 3\cos\theta$

3

−3

↑ shift up 3 units

## Cartesian Plot

$r = 3 + 3\cos\theta$

All Forwards

6

3

+        +

Max    $\frac{\pi}{2}$    π    $\frac{3\pi}{2}$    2π Max

Min

$r$ ·    6        6
          3        3
             0

## Polar Plot

$r = 3 + 3\cos\theta$

3

0        6    θ = 0
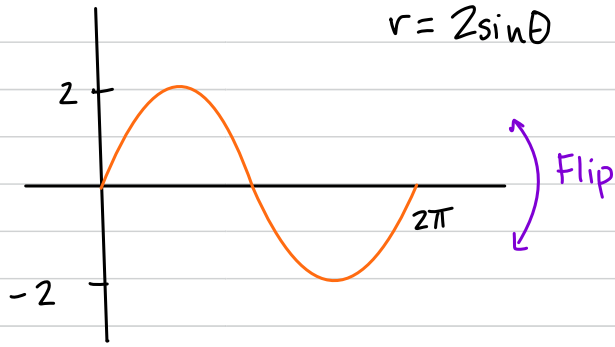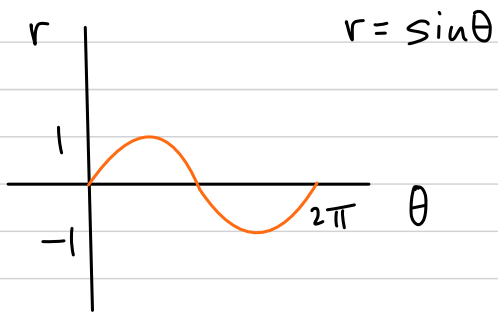
3
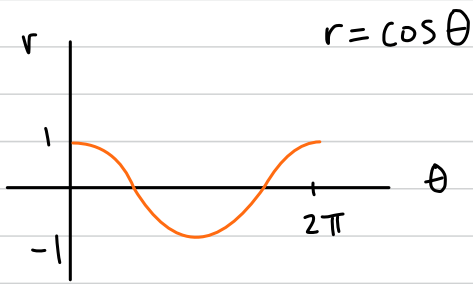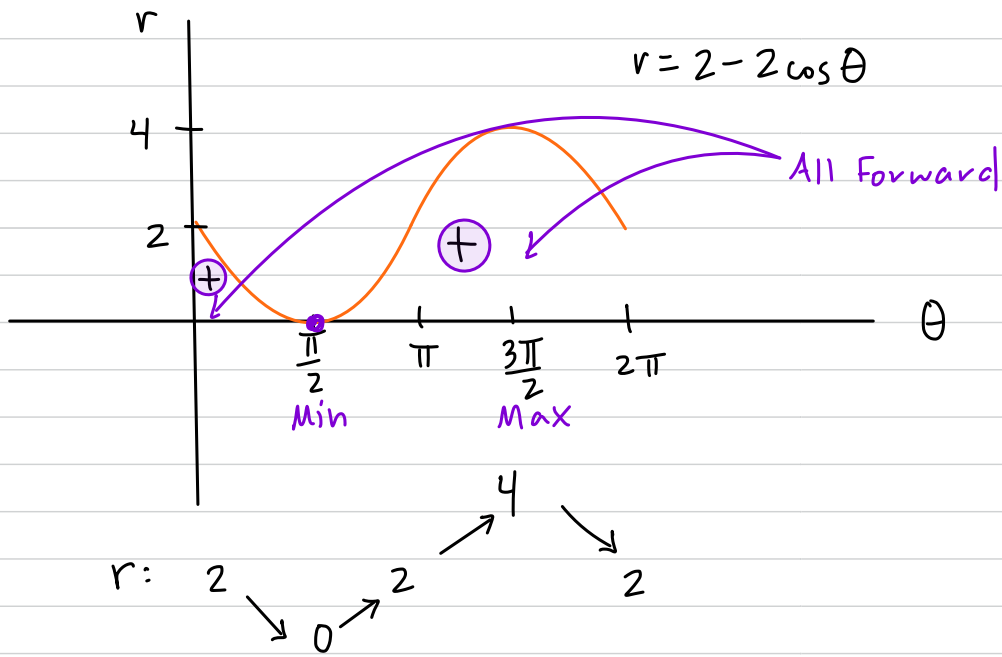
Note: 3-6-3 Cardioid closes one Loop as θ ranges from 0 to 2π
Then retraces same Cardioid, repeating every 2π cycle.

4   $r = 2 - 2\sin\theta$
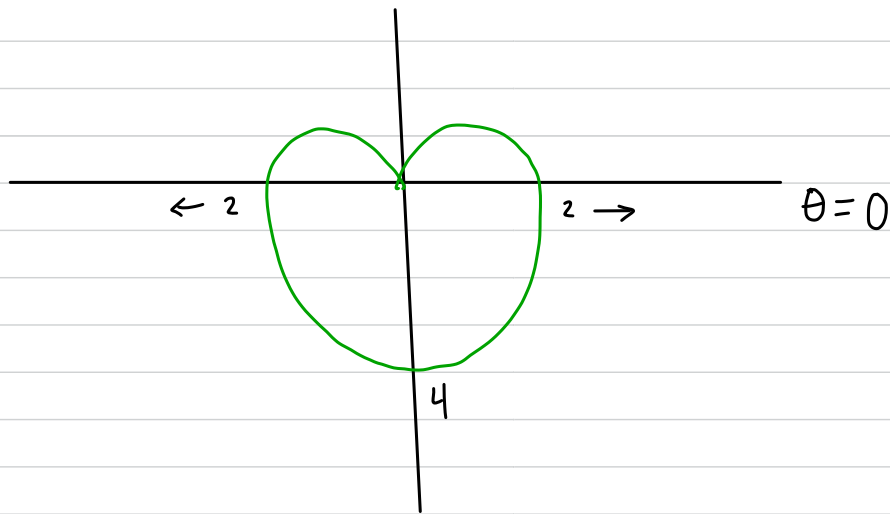
**r = sinθ**

r

1

−1

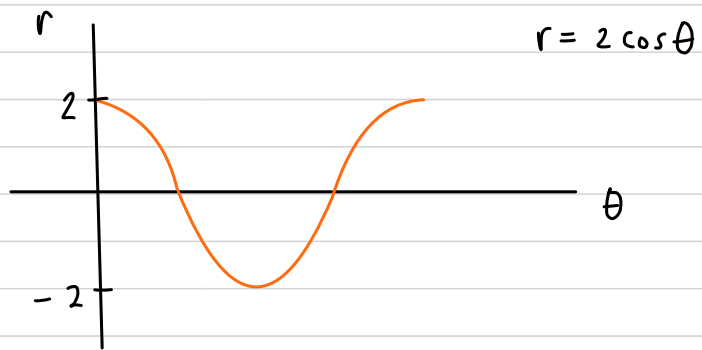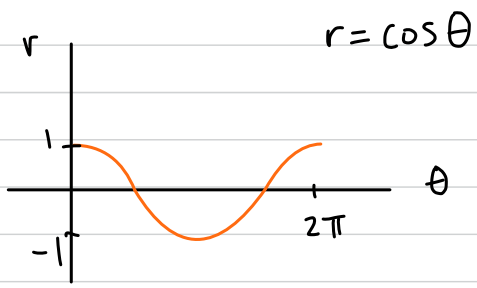2π   θ

**r = 2sinθ**

2

−2

2π

Flip

**r = −2sinθ**

2

−2

π   2π

↑shift up
  2 units

Cartesian Plot

r

4

2

$r = 2 - 2\cos\theta$

All Forward

$\frac{\pi}{2}$   π   $\frac{3\pi}{2}$   2π   θ

Min      Max

                        4
                   ↗  ↘
r:   2   →   2  →      2
      ↘  0 ↗

Polar Plot

← 2          2 →      θ = 0

4

Note: Like a 2-4-2 Cardioid "flipped"

**5.** $r = 1 + 2\cos\theta$

$r = \cos\theta$



$r = 2\cos\theta$

shift up ↑ 1 unit

## Cartesian Plot

$r = 1 + 2\cos\theta$

Forward

$+$    $+$

Backwards

$\pi$

$\dfrac{\pi}{2}$  $\dfrac{2\pi}{3}$   $\dfrac{4\pi}{3}$ $\dfrac{3\pi}{2}$   $2\pi$

$r:$  3 ↘ 1 ⋯ 0 ↘ −1 ↗ 0 ⋯ 1 ↗ 3
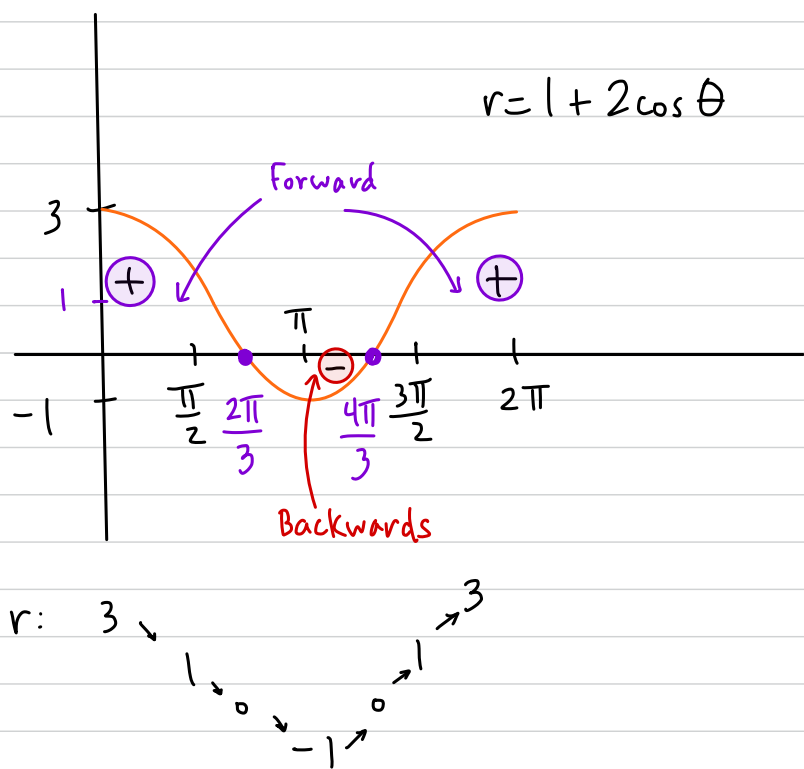
$r = 1 + 2\cos\theta \overset{\text{Set}}{=} 0$
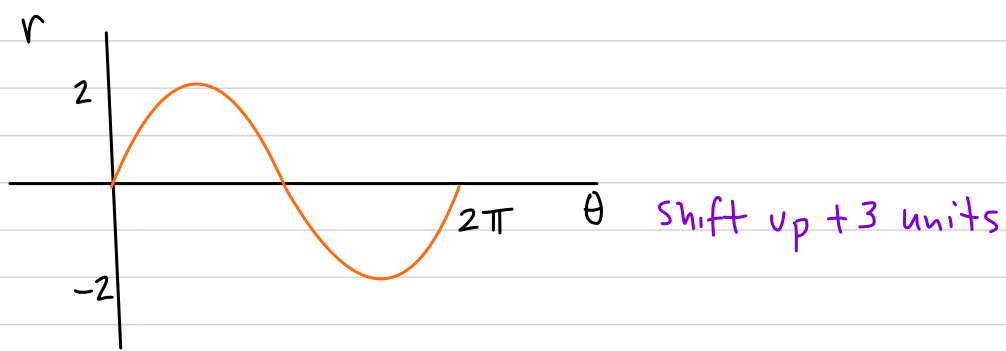
$\quad\hookrightarrow 2\cos\theta = -1$

$\qquad\cos\theta = -\dfrac{1}{2}$

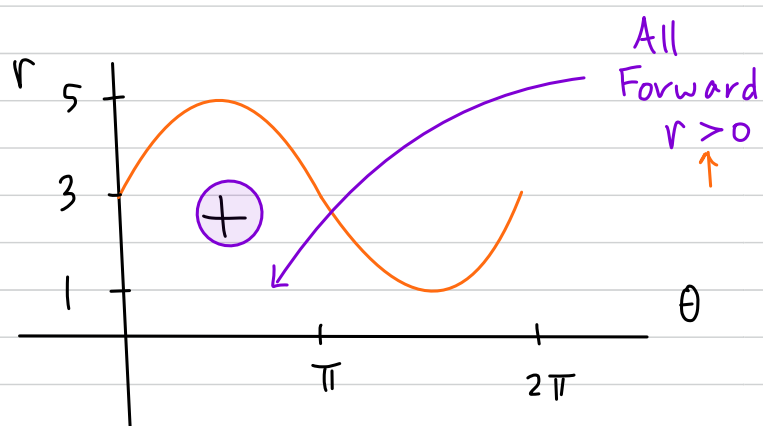$\qquad\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$   zeroes, where $r = 0$

## Polar Plot

$\dfrac{2\pi}{3}$

$\dfrac{4\pi}{3}$

Backwards $r \leq 0$
Internal Loop

1

3

Forward $r \geq 0$

6. $r = 3 + 2\sin\theta$

r = sinθ

r   r = sinθ
1
         θ
-1      2π

r
2
         θ
-2      2π

shift up +3 units

## Cartesian Plot

r
5
3
  ⊕
1
      π        2π      θ

All
Forward
r > 0
↑
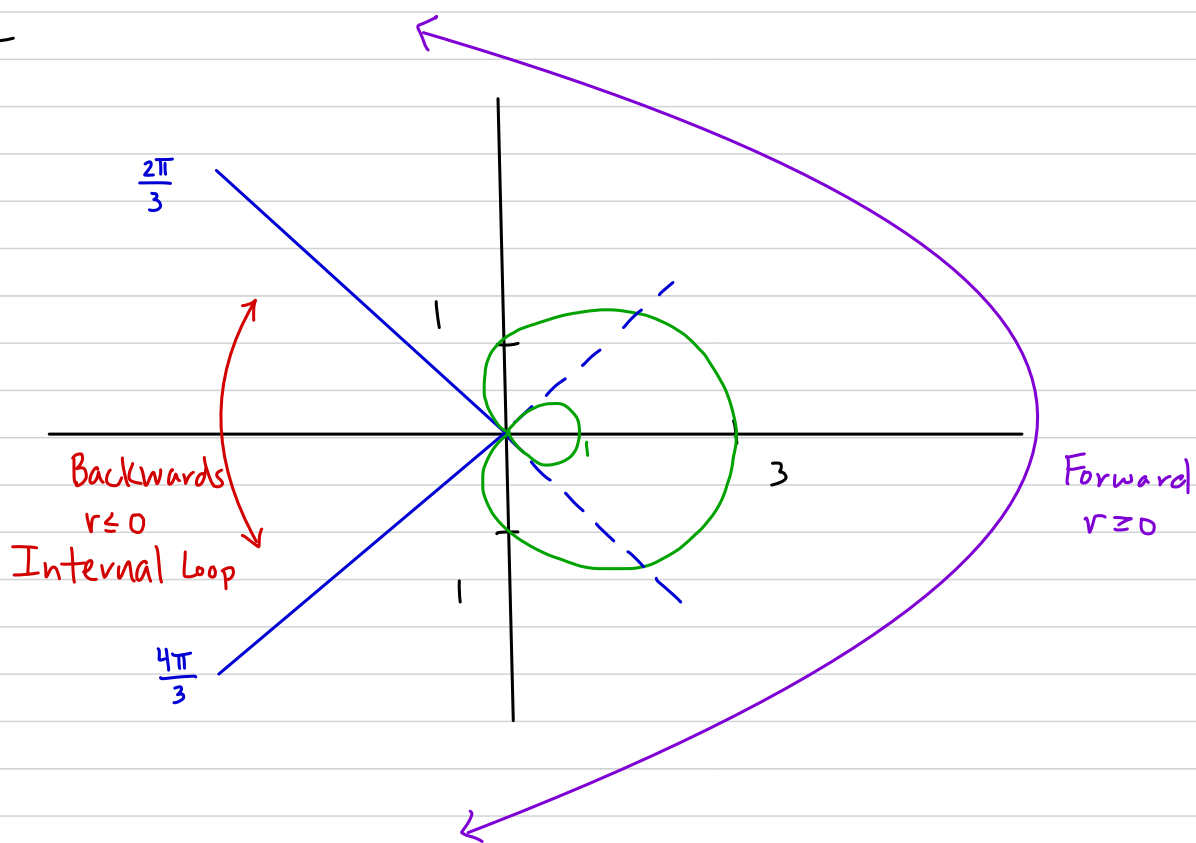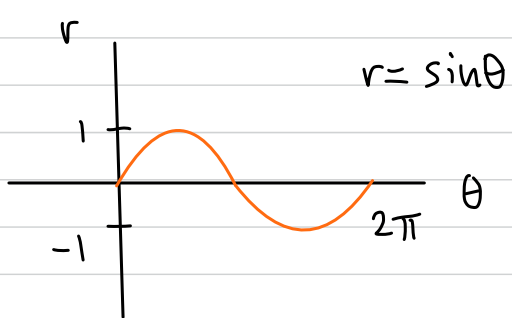
Note: no r=0 or Negative r values

r.    3      ⁵      3        ₃
                        1

## Polar Plot

5

3        3

1
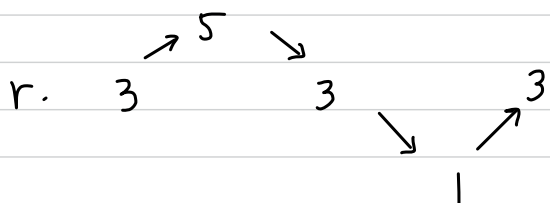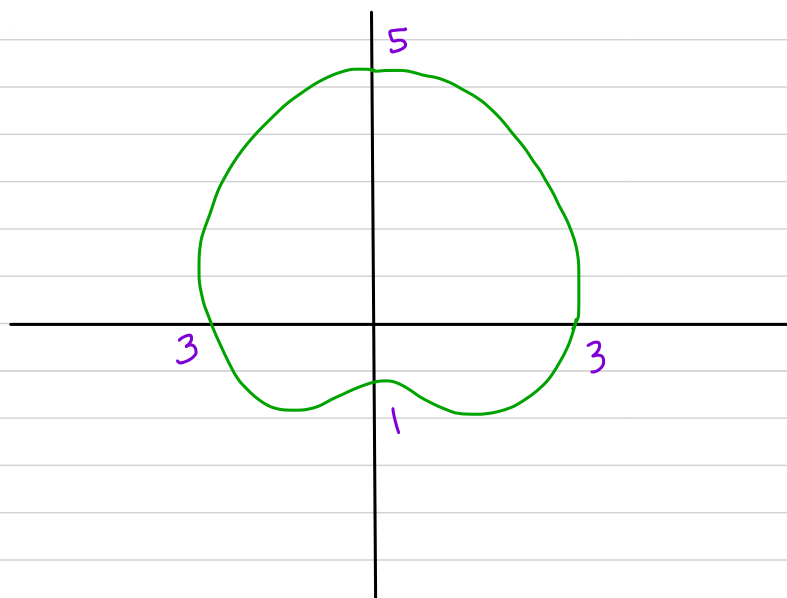
Note: No Cusp, no r=0

7. $r = 3 \sin(2\theta)$

r = sinθ

r = sin(2θ)
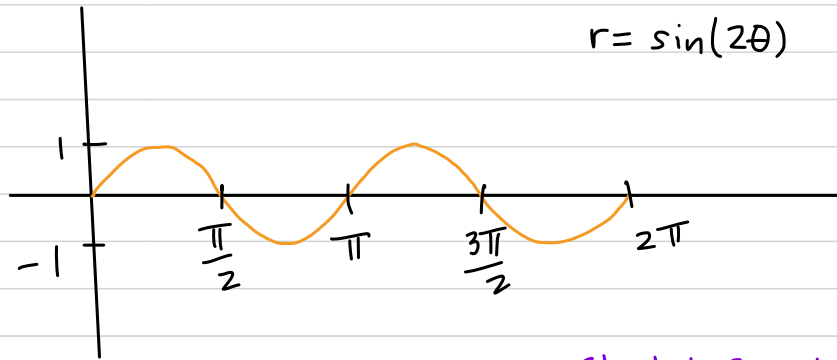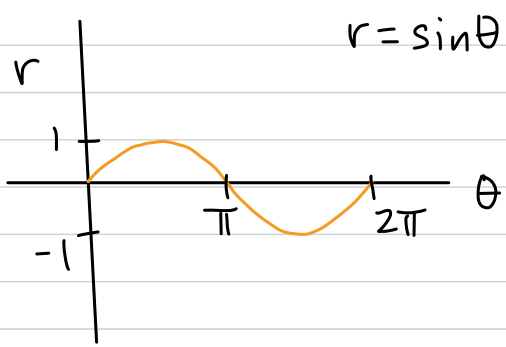
Stretch 3 units
in output

## Cartesian Plot

$r = 3\sin(2\theta)$

Forward

Backwards

Max F   Max B   Max F   Max B

r:   0 → 3 → 0 → -3 → 0 → 3 → 0 → -3 → 0

## Polar Plot

B   $\frac{3\pi}{4}$          F   $\frac{\pi}{4}$

3          3

3          3

$\frac{5\pi}{4}$   F          B   $\frac{7\pi}{4}$