• Please see the course webpage for the answer key.

Find the MacLaurin Series representation for each of the following functions. State the Radius of Convergence for each series. You answer should be in sigma notation $\sum\limits_{n=0}^{\infty}$.

1. $\dfrac{x^2}{1+5x}$

2. $x^7 \sin(x^2)$

3. $x \arctan(3x)$

4. $x^4 e^{-x^3}$

5. $x^3 \ln(1+x^3)$

6. $4x^2 \cos(4x)$

7. $\dfrac{x^3}{4+x}$

8. $\dfrac{d}{dx}\left(\dfrac{x^3}{4+x}\right)$

9. $\displaystyle\int 4x^2 \arctan\left(4x^2\right)\ dx$

10. $\dfrac{d}{dx}\left(x^2 \ln(1+5x)\right)$

11. $\displaystyle\int 5x^3 e^{-5x^4}\ dx$

12. $\dfrac{d}{dx}\left(7x^2 e^{7x}\right)$

13. $\displaystyle\int x^3\ \cos(8x^4)\ dx$

14. $\dfrac{d}{dx}\left(6x^3 \sin(6x^2)\right)$

15. Prove the MacLaurin Series formula for $\arctan x$.

16. Prove the MacLaurin Series formula for $\ln(1+x)$.
Q: Can you think of a second way to prove it?

17. Prove the MacLaurin Series formula for $\sin x$.
Q: Can you think of a second (or third?) way to prove it?