Flash Cards for MacLaurin/Taylor Series   Math 121

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

Interval of Convergence $I = (-\infty, \infty)$

Radius of Convergence $R = \infty$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$$

Interval of Convergence $I = (-\infty, \infty)$

Radius of Convergence $R = \infty$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$= 1 + x + x^2 + x^3 + x^4 + x^5 + \dots$$

Interval of Convergence $I = (-1, 1)$

Radius of Convergence $R = 1$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \dots$$

Interval of Convergence $I = (-\infty, \infty)$

Radius of Convergence $R = \infty$

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \quad \text{(how?)}$$

$$= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} + \dots$$

Interval of Convergence $I = [-1, 1]$

Radius of Convergence $R = 1$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1} \quad \text{(how?)}$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \dots$$

Interval of Convergence $I = (-1, 1]$

Radius of Convergence $R = 1$

Taylor Series
for f at a

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \quad \text{for } |x-a| < R$$

$$= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

MacLaurin Series
for f at $a = 0$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \quad \text{for } |x| < R$$

$$= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$