Math 121-Benedetto

What you need to know for Exam 3

You should know Sections 11.8–11.10, Power Series and their Applications
This exam will not explicitly cover the material from earlier sections, but of course it will still be assumed that you know how to deal with convergence of infinite series, exponentials, logarithms, trig and inverse trig functions, L'Hôpital's rule, substitution, and so on. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- 11.8: Power Series. Know the definition, and remember that it includes not just series of the form $\sum c_n x^n$ but also series like $\sum c_n (x-a)^n$, where a is a constant, called the **center** of the power series. Know how to find the Radius and Interval of convergence of a given power series. Remind yourself what the 3 options are for the (domain) intervals of convergence. See Theorem 3. You will need to recall the works of Sections 11.2-11.6 in order to analyze specific series that arise at each endpoint.

- 11.9: Power Series Representations of Functions. Know how to use power series you already know (like that for $1/(1-x)$, or (in later sections) e^x, $\cos x$, $\sin x$) to find power series for other functions by the following five operations: (1) substituting monomials like $-3x$ or $2x^3$ for x, or (2) multiplying by a polynomial, or (3) adding two power series (centered at the same center), or (4) integrating (usually followed by evaluating at $x = a$ to find the constant C from integration), or (5) differentiating. Know how each of these operations may affect the Radius of Convergence. See the Theorem.

- 11.10: Taylor and MacLaurin Series. Know the following:

  -Computing Taylor/MacLaurin Series using the Definition (Chart Method)

  -Our 6 memorized MacLaurin Series

  -Applications, including new series from known ones, new derivatives, new (indefinite) integrals, new sums (beyond geometric), and estimates for values or (definite) integrals using Alternating Series Estimation Theorem, and finally Limits using Series.

Some things you don't need to know

- Section 11.10: Taylor Series Remainder material
- Sections 11.10: Binomial Series
- All of Section 11.11 except for n^{th} Taylor Polynomials
- Chapter 10 Parametric Equations

Tips

- Know the three possibilities for Intervals of Convergence. If it the finite case, make sure to manually check convergence at the endpoints. Understand why the Ratio Test is inconclusive at the endpoints.

  Remember $\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n = e$ and $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \frac{1}{e}$

- For power series representations: if you're asked to find the MacLaurin or Taylor series (or power series rep) of a function $f(x)$, or even a Taylor polynomial, your first attempt should be to write f in terms of the basic functions $1/(1-x)$, e^x, $\cos x$, $\sin x$ using only the operations that are good for power series (see the 11.9 discussion above). If that doesn't seem to work, then you can try to blast out the Taylor series by taking all those derivatives, using the Definition/*chart method*. It also takes much more time. Finding the MacLaurin series for $x^4\cos x$ is quick if you just multiply the series for $\cos x$ by x^4; but it's a horrendous and time-consuming mess to compute by blasting out all the derivatives using the definition.)

- That is, the 4 main ways to find a function's Power Series
  - Substitution into a known MacLaurin Series
  - Differentiating another Series
  - Integrating another Series
  - Using the Definition/Chart Method

- If using Integration to derive the Series, then we recommend using an expanded Long form of the series to compute +C in the proof.

- If you're asked for the **sum** of a series, on Exam 2 that meant it'd have to be something like a geometric series Now we have another possibility: that you can get that series by plugging in x equals some specific number in some specific power series. For example, $\sum_{n=0}^{\infty}\frac{5^{n-2}}{2^{2n}\cdot n!}$ can be rewritten as $\frac{1}{25}\sum_{n=0}^{\infty}\frac{(5/4)^n}{n!} = \frac{1}{25}e^{5/4}$. Similarly, $\sum_{n=0}^{\infty}\frac{(-1)^n 7^n}{n!} = e^{-7}$.