Math 121, Section(s) 01, 02, Fall 2024

Homework #16

Due **Friday, November 15th** in Gradescope by 11:59 pm ET

Goal: Exploring more of the Relationship between Power Series and functions, including Differentiation and Integration of Power Series. Also *substitution* into a known MacLaurin Series. Also SUMS which are not Geometric.

Find the Series Representation for the following functions using *substitution* and determine the Radius of Convergence R. Simplify.

1. $\dfrac{1}{1+x^2}$
2. $\dfrac{x^2}{x^4+16}$
3. $x^3 \cos\left(x^2\right)$
4. $5x^2 \sin(5x)$
5. $\dfrac{d}{dx}\left(x^3 \arctan(7x)\right)$
6. $\displaystyle\int x^3 \arctan(7x)\, dx$
7. $\dfrac{d}{dx} x^2 \ln(1+6x)$
8. $\displaystyle\int x^4 e^{-x^3}\, dx$

9. Find the Series Representation for $f(x) = \dfrac{1}{(1+x)^2}$

Hint: $$\frac{1}{(1+x)^2} = \frac{d}{dx}\left(-\overset{PS?}{\frac{1}{1+x}}\right) = \dots$$

10. Prove the Power Series Representation formula for $\arctan x$, as shown in class. Yes, show that $C = 0$.

11. Find Series Representation for $\ln(5-x)$. Solve for C and the Radius R.

Hint: $$\ln(5-x) = \int \frac{-1}{5-x}\, dx = \int \frac{-1}{5\left(1-\frac{x}{5}\right)}\, dx = -\frac{1}{5}\int \overset{PS?}{\frac{1}{1-\frac{x}{5}}}\, dx = \dots$$

12. Find the MacLaurin Series for $f(x) = e^{-2x}$ using two different methods.

First, using the *Definition* of the MacLaurin Series ("Chart Method").

Second, use Substitution into a known series. Your answers should be in Sigma notation.

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13. You do **not** need to state the Radius. Answers should be in Sigma notation $\sum_{n=0}^{\infty}$ here.

You may use the fact that $\boxed{\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}}$ without extra justification.

(a) Use the Definition ("Chart Method") to compute the MacLaurin Series for $F(x) = \cos x$.

(b) Use Differentiation to compute the Series for $F(x) = \cos x$.

(c) Use Integration to compute the Series for $F(x) = \cos x$.

Hints: yes, you should solve for $+C$. yes, C should equal 1. Show why $C = 1$.

Find the Sum of each of the following Series, which do converge.

14. $\sum_{n=0}^{\infty} \frac{7^n}{n!}$

15. $\sum_{n=0}^{\infty} \frac{(-1)^n\ 5^n}{n!}$

16. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$

17. $\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n}}{6^{2n}\ (2n)!}$

18. $\sum_{n=0}^{\infty} \frac{3^n}{5^n\ n!}$

19. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

20. $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \ldots$

21. $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \ldots$

REGULAR OFFICE HOURS

Sunday 6:00–9:00 pm TAs Natalie/Oscar, SMUDD 207

Monday: 12:00–3:00 pm

6:00–9:00 pm TAs Aaron/Oscar, SMUDD 207

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Gretta, SMUDD 207

Wednesday: 1:00-3:00 pm

7:30–9:00 pm TA Natalie, SMUDD 207

Thursday: none for Professor

extras may be added, TBD weekly

6–9:00 pm TAs Gretta/DJ, SMUDD 207

Friday: 12:00–3:00 pm

6:00–9:00 pm TAs Aaron/DJ, SMUDD 207

Pay careful attention to details here.
Manipulating power series requires a balance
of memory and technical skill.