Math 121, Benedetto, Spring 2026

# OPTIONAL BONUS HOMEWORK

Due **Wednesday, April 29th** in Gradescope by 11:59 pm

Directions: You may attempt the following bonus questions. A correct, complete, and fully detailed solution will get you +1 Bonus point each, up to 5 total, on the upcoming Final Exam. You may **not** work with any other people, and the **only** reference sources you may use are your notes and course webpage. **No** other on-line sources or calculators. **No** academic dishonesty.

1. Estimate $\sin\left(\frac{1}{2}\right)$ with error less than $\frac{1}{500,000}$. Simplify to a common denominator.

2. Create a Power Series with Interval of Convergence $\left[-\frac{7}{6}, \frac{1}{5}\right]$. Continue on to justify that this series satisfies this challenge.

3. Find the sum value for $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\pi^{2n+3}}{(2n+5)!}$

4. Find the sum value for $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n}}{(2n+1)3^n}$

5. Find the sum value for $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n}\pi^{2n-1}}{2!(256)^{\frac{n}{4}}(2n)!}$

6. Find the Power Series for $\frac{x^7}{(1-3x)^2}$.

7. First, compute the Degree 2 Taylor polynomial $T_2(x)$ for $f(x) = \sqrt{x}$ centered at $a = 16$. Then use this polynomial to Estimate $\sqrt{15}$ by computing $T_2(15)$. Simplify answer to a fraction.

8. Let $f(x) = x^2 \sin(x^3)$. Compute the $22^{nd}$ and $23^{rd}$ Derivatives *evaluated at 0*, that is $f^{[22]}(0)$ and $f^{[23]}(0)$. Do not simplify your answers. Do not compute the derivatives manually.

9. Find the sum value for $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n}(2n+5)\pi^{2n+1}}{(2n+1)!}$

10. Find the sum value $\displaystyle\sum_{n=0}^{\infty} \frac{n^2(\ln 3)^n}{n!}$

11. Create a limit $\displaystyle\lim_{x\to 0} \frac{G(x)}{H(x)}$ which equals $-\frac{5}{12}$ and requires 3 applications of L'Hôpital's Rule. The expression $\frac{G(x)}{H(x)}$ must include at least **two** of $\sin x$, $\ln(1+x)$, $\cos x$, $\arctan x$, and $e^x$. Continue on to compute the Limit using Series, and then also using the three L'H Rules.