Math 121 Midterm Exam #3 May 6-9, 2021

Due Sunday, May 9, in Gradescope by 11:59 pm ET

- This is an *Open Notes* Exam. You can use materials, homeworks problems, lecture notes, etc. that you manually worked on.
- There is **NO** *Open Internet* allowed. You can only access our Main Course Webpage.
- You are not allowed to work on or discuss these problems with anyone. You can ask a few small, clarifying, questions about instructions in Office Hours, but these problems will not be solved.
- Submit your final work in Gradescope in the Exam 3 entry.
- Please *show* all of your work and *justify* all of your answers. No Calculators.

1. [14 Points] Find the **Interval** and **Radius** of Convergence for the following power series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \, (6x+1)^n}{(6n+1)^2 \cdot 7^n}$$

2. [6 Points] Design a Power Series which is convergent only at $x = 5$. Once you create your series, then proceed to justify that the Interval of Convergence is indeed $I = \{5\}$.

3. [6 Points] Design a Power Series which is convergent only on the open interval $5 < x < 7$. Once you create your series, then proceed to justify that the Interval of Convergence is indeed $I = (5, 7)$.

4. [10 Points] Find the MacLaurin Series for each of the following functions. **State** the Radius of Convergence for each series. Your answers should all be in sigma notation $\sum_{n=0}^{\infty}$ here. Simplify.

(a) $\dfrac{x^2}{4+x}$ (b) $6x^4 \arctan(6x)$

5. [10 Points] Your answers should all be in sigma notation $\sum_{n=0}^{\infty}$ here. Simplify.

(a) Use MacLaurin Series to compute $\dfrac{d}{dx}\left(5x^2 e^{-x^3}\right)$.

(b) Use MacLaurin Series to compute $\displaystyle\int x^3 \sin\left(8x^4\right)\, dx$.

6. [7 Points] Use MacLaurin Series to Estimate $\cos\left(\frac{1}{2}\right)$ with error less than $\frac{1}{10000}$.

Tips: $6! = 720$ and $(720)\cdot(64) = 46,080$ and $(48)\cdot 8 = 384$

7. [24 Points] Find the **sum** for each of the following series (which do converge). Simplify.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n+1}}{9^{n-1}\ (2n+1)!}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)\ 3}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)\ 3^n}$ hint: $3 = \left(\sqrt{3}\,\right)^2$

(d) $-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \ldots$ (e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\pi^{2n}}{4!\ (2n)!}$ (f) $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \ldots$

8. [15 Points] Do not just write a formula. You do **not** need to state the Radius. Your answers should all be in Sigma notation $\displaystyle\sum_{n=0}^{\infty}$ here.

You may use the fact that $\boxed{\cos x = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}}$ without extra justification.

(a) Demonstrate one method to compute the MacLaurin Series for $F(x) = \sin x$.

(b) Demonstrate a second, **different**, method to compute the MacLaurin Series for $F(x) = \sin x$.

(c) Demonstrate a third, **different**, method to compute the MacLaurin Series for $F(x) = \sin x$.
Hint: yes, you should solve for $+C$.

9. [8 Points] Consider the Parametric Curve given by $x = (\arctan t) - t$ and $y = 2\sinh^{-1} t$.

Compute the Arclength of this parametric curve for $0 \le t \le \sqrt{3}$. Hint: $\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{1+x^2}}$

OPTIONAL, Just for Fun you can turn it in if you want to

Compute $\displaystyle\lim_{x\to 0} \frac{xe^x - \sin x}{\ln(1+x) - \arctan x}$ in two ways: using L'Hôpital's Rule and then using Series.