



- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- Numerical answers such as  $\sin\left(\frac{\pi}{6}\right)$ ,  $4^{\frac{3}{2}}$ ,  $\arctan(\sqrt{3})$ ,  $e^{\ln 4}$ ,  $\ln(e^7)$ , or  $e^{3\ln 3}$  should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

**1.** [27 Points] Compute the following **Improper** integrals. Simplify all answers. Justify.

(a)  $\int_0^e \ln x \, dx$

(b)  $\int_0^{e^4} \frac{1}{x [16 + (\ln x)^2]} \, dx$

(c)  $\int_3^4 \frac{x+1}{x^2-4x+3} \, dx$  Free Partial Fractions  $\frac{x+1}{(x-3)(x-1)} = \frac{2}{x-3} - \frac{1}{x-1}$

**2.** [14 Points]

Use the **Absolute Convergence Test** to show  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 4n + 7}$  **Converges**.

**IMPORTANT:** You are required to use the **Integral Test on the Absolute Series** in this problem.

Note: You do **not** have to check the 3 pre-conditions for the Integral Test.

**3.** [8 Points] **CREATE** a Series that **Converges by the Comparison Test**.

You cannot choose just a  $p$ -series. You can *compare* with a  $p$ -series. TIP: Try to choose a simple series.

Continue on to prove that the Series Converges by the Comparison Test.

**4.** [21 Points] Determine whether each of the given series **Converges** or **Diverges**. Name any Convergence Test(s) you use, and justify all of your work.

(a)  $-6 - \frac{6}{2} - \frac{6}{3} - \frac{6}{4} - \frac{6}{5} - \dots$       (b)  $\sum_{n=1}^{\infty} \frac{(2026)!}{n^6} + \frac{(-2)^n}{6^{2n}}$        $\sum_{n=2}^{\infty} \frac{n^6}{\ln(2n+1)}$

**5.** [30 Points] Determine whether each of the given series is **Absolutely Convergent**, **Conditionally Convergent**, or **Divergent**. Name any Convergence Test(s) you use, and justify all of your work.

(a)  $\sum_{n=1}^{\infty} (-1)^n \left( \frac{n^2 + 6}{n^6 + 2} \right)$

(b)  $\sum_{n=1}^{\infty} \frac{(-1)^n (2n+1)!}{n^n (n!)}$

(c)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{6n+2}$

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## OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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**OPTIONAL BONUS #1** Create a Series which contains at least  $n^n$  and  $(2n)!$  and is Divergent by the Ratio Test and has the Ratio Test Limit  $L = \frac{28e^3}{\pi^2}$

Continue on to Prove that the Series Diverges by the Ratio Test with that specific  $L > 1$ .