Math 121 Midterm Exam #2 April 11, 2022

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [36 Points] Compute the following **Improper** Integrals. Justify your work.

(a) $\int_{-4}^{-3} \frac{6}{x^2 + 2x - 8} dx$ You can use this **free** Partial Fractions fact:

$$\frac{6}{(x-2)(x+4)} = \frac{1}{x-2} - \frac{1}{x+4}$$

(b)
$$\int_{-\infty}^{0} \frac{6}{x^2 + 2x + 4} dx$$

(c) $\int_0^1 \ln x \, dx$ You can use this **free** fact $\int \ln x \, dx = x \ln x - x + C$

(d)
$$\int_0^{e^5} \frac{1}{x \left[25 + (\ln x)^2\right]} dx$$

2. [20 Points] Determine whether each of the given series Converges or Diverges. Name any convergence test(s) you use, and justify all of your work.

(a)
$$\sum_{n=1}^{\infty} \frac{1}{6^{2n}} + \frac{\ln(2022)}{n^6}$$

(b) $\sum_{n=1}^{\infty} \frac{\arctan n}{2022}$
(c) $\sum_{n=2}^{\infty} \frac{n^6}{\ln(n+2022)}$

3. [8 Points] Name any convergence test(s) you use, and justify all of your work.

Use the Absolute Convergence Test to prove that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^6 + 2022}$ Converges.

4. [36 Points] In each case Determine whether the given series is Absolutely Convergent, Conditionally Convergent, or Diverges. Name any convergence test(s) you use, and justify all of your work.

(a)
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 6}{n^6 + 2}$$
 (b) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 6^n \cdot n!}{n^6 \cdot n^n}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{6n + 2022}$

OPTIONAL BONUS

OPTIONAL BONUS #1 Prove that the sequence $\left\{\frac{(\ln n) \cdot 2^n \cdot (n!)^2}{n^{2n} \cdot (3n)!}\right\}_{n=1}^{\infty}$ Converges.