



Math 121 Exam #2 Nov 1-Nov 3



Due Sunday, November 3, 2024 in Gradescope by 11:59 pm

- This is **NOT** an Open Notes Exam.
- Do **NOT** access any materials, homework problems, lecture notes...
- There is **NO** *Open Internet* access allowed. Do **NOT** use any online/electronic sources.
- You are **NOT** allowed to discuss these problems with anyone, including Math Fellows.
- You **MAY** only use one 5x7 Cheat Sheet card and the Flash Card sheet.
- Submit your final work in Gradescope in the Exam 2 entry.
- Please *show* all of your work and *justify* all of your answers. No Calculators.
- Submit a Final Draft, that is, a very neat and persuasive solution set.

1. [24 Points] Compute the following **Improper** integrals. Simplify all answers. Justify.

$$(a) \int_0^e x^3 \ln x \, dx \quad (b) \int_{-\infty}^{-9} \frac{7}{x^2 + 4x + 53} \, dx \quad (c) \int_{-5}^{-4} \frac{7-x}{x^2 + 4x - 5} \, dx$$

2. [8 Points] Use the Integral Test to determine if $\sum_{n=1}^{\infty} \frac{\ln n}{n^7}$ **Converges** or **Diverges**.

Note: You do **not** have to check the 3 pre-conditions for the Integral Test.

3. [36 Points] Determine whether each of the given series **Converges** or **Diverges**. Name any Convergence Test(s) you use, and justify all of your work.

$$(a) \sum_{n=2}^{\infty} \frac{e^{3n}}{7 \ln n} \quad (b) \sum_{n=1}^{\infty} \frac{1}{(3n+7)!} \quad (c) -\frac{3}{\sqrt{1}} - \frac{3}{\sqrt{2}} - \frac{3}{\sqrt{3}} - \frac{3}{\sqrt{4}} - \frac{3}{\sqrt{5}} - \frac{3}{\sqrt{6}} - \dots$$

$$(d) \sum_{n=1}^{\infty} \frac{\ln 3}{n^7} + \frac{(-3)^n}{7^{2n+1}} \quad (e) \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{3}{n^7} \right) \quad (f) \sum_{n=2}^{\infty} \left(1 - \frac{7}{n^3} \right)^{n^3}$$

4. [6 Points] Use the Absolute Convergence Test to show that $\sum_{n=1}^{\infty} \frac{(-1)^n}{(3n+7)^7}$ Converges.

5. [26 Points] Determine whether each of the given series is **Absolutely Convergent**, **Conditionally Convergent**, or **Divergent**. Name any Convergence Test(s) you use, and justify all of your work.

(a) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^3 + 7}{n^7 + 3} \right)$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n (2n+1)! 3^n}{n^7 (n!) n^n}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+7}$

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

BONUS 1: Prove that the **Sequence** $\left\{ \frac{(\ln(\ln n)) \cdot 5^n \cdot (n!)^3 (2n)!}{n^{2n} \cdot (3n)!} \right\}_{n=1}^{\infty}$ Converges.

BONUS 2: Compute the **Sum** value of the Series $\sum_{n=1}^{\infty} \frac{e^{2n+2} - e^{2n}}{(e^{2n} + 1)(e^{2n+2} + 1)}$