



Math 121 Exam 1

February 20, 2026



- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\arctan(\sqrt{3})$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. Limits [30 Points total] Evaluate each of the following. Please justify/simplify.

(a) Show that $\lim_{x \rightarrow 0} \frac{\ln(1+5x) - 5x}{\arcsin(3x) + e^{-3x} - 1} = \boxed{-\frac{25}{9}}$

(b) Show that $\lim_{x \rightarrow 0^+} x^3 \ln x = \boxed{0}$

(c) Show that $\lim_{x \rightarrow \infty} \left(1 - \arctan\left(\frac{3}{x^4}\right)\right)^{x^4} = \boxed{e^{-3}}$

Integrals [36 Points total] Compute each of the following Definite Integrals. Justify.

2. Show that $\int_{-3}^3 \sqrt{9-x^2} dx = \boxed{\frac{9\pi}{2}}$

3. Show that $\int_0^{\ln \sqrt{3}} \frac{e^{2x}}{3+e^{4x}} dx = \boxed{\frac{\pi}{12\sqrt{3}}}$

4. Show that $\int_1^e x^3 \ln x dx = \boxed{\frac{1+3e^4}{16}}$

More Integrals [34 Points total] Compute each of the following Indefinite Integrals. Justify.

5. Compute $\int x^4 \arcsin x dx$

6. Compute $\int \frac{1}{(9+x^2)^{\frac{7}{2}}} dx$ Hint: $3^6 = 729$

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\int \frac{xe^x}{\sqrt{e^x - 1}} dx$