

Exam 1 Spring 2025 Answer Key

$$\begin{aligned}
 1(a) \quad \lim_{x \rightarrow 0} \frac{\cos(3x) - \arctan(2x) + 2x - 1}{e^{-4x} - 1 + 4x} &= \lim_{x \rightarrow 0} \frac{-3\sin(3x) - \frac{2}{1+(2x)^2} + 2}{-4e^{-4x} + 4} \\
 &= \lim_{x \rightarrow 0} \frac{-3\sin(3x) - 2(1+4x^2)^{-1} + 2}{-4e^{-4x} + 4} \\
 &= \lim_{x \rightarrow 0} \frac{-9\cos(3x) + 2(1+4x^2)^{-2} \cdot (8x)}{16e^{-4x}} \\
 &= \lim_{x \rightarrow 0} \frac{-9\cos(3x) + \frac{16x}{(1+4x^2)^2}}{16e^{-4x}} = \frac{-9}{16}
 \end{aligned}$$

$$1(b) \quad \lim_{x \rightarrow 0^+} \sqrt{x} \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{\sqrt{x}}} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-\frac{1}{2x^{3/2}}} = \lim_{x \rightarrow 0^+} -2\sqrt{x} = 0$$

$$\frac{d}{dx} x^{-1/2} = -\frac{1}{2} x^{-3/2}$$

$$\begin{aligned}
 1(c) \quad \lim_{x \rightarrow \infty} \left(1 - \arcsin\left(\frac{2}{x^6}\right)\right)^{x^6} &= e^{\lim_{x \rightarrow \infty} \ln\left(\left(1 - \arcsin\left(\frac{2}{x^6}\right)\right)^{x^6}\right)} \\
 &= e^{\lim_{x \rightarrow \infty} x^6 \cdot \ln\left(1 - \arcsin\left(\frac{2}{x^6}\right)\right)} \\
 &= e^{\lim_{x \rightarrow \infty} \frac{\ln\left(1 - \arcsin\left(\frac{2}{x^6}\right)\right)}{\frac{1}{x^6}}} \\
 &= e^{\lim_{x \rightarrow \infty} \frac{\frac{-1}{1 - \arcsin\left(\frac{2}{x^6}\right)} \cdot \frac{-1}{\sqrt{1 - \left(\frac{2}{x^6}\right)^2}} \cdot \left(\frac{-12}{x^7}\right)}{-6x^{-7}}} \\
 &= e^{1 \cdot (-1) \cdot 2} = e^{-2}
 \end{aligned}$$

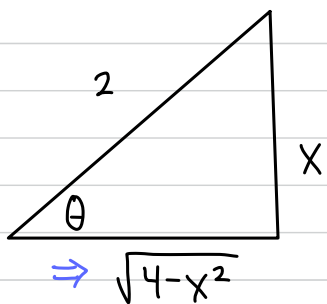
$$2. \int_{-2}^2 \sqrt{4-x^2} dx = \int_{x=-2}^{x=2} \sqrt{4-4\sin^2\theta} \cdot 2\cos\theta d\theta = 4 \int_{x=-2}^{x=2} \cos^2\theta d\theta$$

$\sqrt{4(1-\sin^2\theta)}$
 $\sqrt{4\cos^2\theta}$

Trig. Sub $x = 2\sin\theta$
 $dx = 2\cos\theta d\theta$

$$\sin\theta = \frac{x}{2} \Rightarrow \theta = \arcsin\left(\frac{x}{2}\right)$$

$$= 4 \int_{x=-2}^{x=2} \frac{1+\cos(2\theta)}{2} d\theta = \frac{4}{2} \int_{x=-2}^{x=2} 1+\cos(2\theta) d\theta$$



$$= 2 \left(\theta + \frac{\sin(2\theta)}{2} \right) \Big|_{x=-2}^{x=2} = 2 \left(\arcsin\left(\frac{x}{2}\right) + \frac{x}{2} \frac{\sqrt{4-x^2}}{2} \right) \Big|_{-2}^2$$

$$= 2 \left(\arcsin\left(\frac{2}{2}\right) + \left(\frac{2}{2}\right) \frac{\sqrt{4-4}}{2} - \left(\arcsin\left(\frac{-2}{2}\right) + \left(\frac{-2}{2}\right) \frac{\sqrt{4-4}}{2} \right) \right)$$

$$= 2 \left(\frac{\pi}{2} + \frac{\pi}{2} \right) = 2\pi$$

$$3. \int_0^{\ln\sqrt{3}} \frac{e^x}{\sqrt{4-e^{2x}}} dx = \int_0^{\ln\sqrt{3}} \frac{e^x}{\sqrt{4-(e^x)^2}} dx = \int_1^{\sqrt{3}} \frac{1}{\sqrt{4-u^2}} du = \arcsin\left(\frac{u}{2}\right) \Big|_1^{\sqrt{3}}$$

$u = e^x$
 $du = e^x dx$

$x=0 \Rightarrow u=e^0=1$
 $x=\ln\sqrt{3} \Rightarrow u=e^{\ln\sqrt{3}}=\sqrt{3}$

$$= \arcsin\left(\frac{\sqrt{3}}{2}\right) - \arcsin\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

$$4. \int_e^{e^3} \frac{1}{x(3+(\ln x)^2)} dx = \int_1^3 \frac{1}{3+u^2} du = \frac{1}{\sqrt{3}} \arctan\left(\frac{u}{\sqrt{3}}\right) \Big|_1^3$$

$u = \ln x$
 $du = \frac{1}{x} dx$

$x=e \Rightarrow u=\ln e=1$
 $x=e^3 \Rightarrow u=\ln e^3=3$

$$= \frac{1}{\sqrt{3}} \left(\arctan\left(\frac{3}{\sqrt{3}}\right) - \arctan\left(\frac{1}{\sqrt{3}}\right) \right)$$

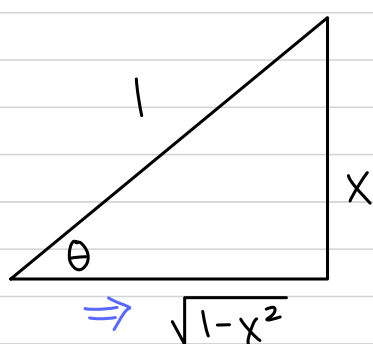
$$= \frac{1}{\sqrt{3}} \left(\frac{\pi}{3} - \frac{\pi}{6} \right) = \frac{\pi}{6\sqrt{3}}$$

$$5. \int x \arcsin x \, dx = \frac{x^2}{2} \arcsin x - \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} \, dx = \frac{x^2}{2} \arcsin x - \frac{1}{2} \int \frac{\sin^2 \theta}{\sqrt{1-\sin^2 \theta}} \cdot \cos \theta \, d\theta$$

$$\begin{aligned} u &= \arcsin x & dv &= x \, dx \\ du &= \frac{1}{\sqrt{1-x^2}} \, dx & v &= \frac{x^2}{2} \end{aligned}$$

$$\begin{aligned} x &= \sin \theta \\ dx &= \cos \theta \, d\theta \end{aligned}$$

$$\theta = \arcsin x$$



$$= \frac{x^2}{2} \arcsin x - \frac{1}{2} \int \sin^2 \theta \, d\theta$$

$$= \frac{x^2}{2} \arcsin x - \frac{1}{2} \int \frac{1-\cos(2\theta)}{2} \, d\theta$$

$$= \frac{x^2}{2} \arcsin x - \frac{1}{4} \left(\theta - \frac{\sin(2\theta)}{2} \right) + C$$

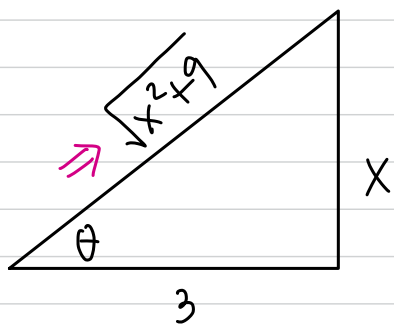
$$= \frac{x^2}{2} \arcsin x - \frac{1}{4} \left(\arcsin x - x \sqrt{1-x^2} \right) + C$$

$$\text{OR} // = \frac{x^2}{2} \arcsin x - \frac{1}{4} \arcsin x + \frac{1}{4} x \sqrt{1-x^2} + C$$

$$6. \int \frac{1}{(9+x^2)^{7/2}} \, dx = \int \frac{1}{(\sqrt{9+x^2})^7} \, dx = \int \frac{1}{(\sqrt{9+9\tan^2 \theta})^7} \cdot 3 \sec^2 \theta \, d\theta$$

$$\begin{aligned} x &= 3 \tan \theta \\ dx &= 3 \sec^2 \theta \, d\theta \end{aligned}$$

$$\tan \theta = \frac{x}{3}$$



$$= \int \frac{1}{3^7 \cdot \sec^7 \theta} \cdot 3 \sec^2 \theta \, d\theta = \frac{1}{3^6} \int \frac{1}{\sec^5 \theta} \, d\theta$$

$$= \frac{1}{729} \int \cos^5 \theta \, d\theta = \frac{1}{729} \int \cos^4 \theta \cdot \cos \theta \, d\theta = \frac{1}{729} \int (1-\sin^2 \theta)^2 \cdot \cos \theta \, d\theta$$

$$= \frac{1}{729} \int (1-u^2)^2 \, du = \frac{1}{729} \int 1 - 2u^2 + u^4 \, du = \frac{1}{729} \left(u - \frac{2}{3} u^3 + \frac{u^5}{5} \right) + C$$

$$\begin{aligned} u &= \sin \theta \\ du &= \cos \theta \, d\theta \end{aligned}$$

$$= \frac{1}{729} \left(\sin \theta - \frac{2}{3} \sin^3 \theta + \frac{1}{5} \sin^5 \theta \right) + C$$

$$= \frac{1}{729} \left(\frac{x}{\sqrt{x^2+9}} - \frac{2}{3} \left(\frac{x}{\sqrt{x^2+9}} \right)^3 + \frac{1}{5} \left(\frac{x}{\sqrt{x^2+9}} \right)^5 \right) + C$$

$$7. \int \ln(x^2+9) dx = \int \ln(x^2+9) \cdot 1 dx = x \ln(x^2+9) - 2 \int \frac{x^2+9-9}{x^2+9} dx \quad \text{slip-in/slip-out}$$

$u = \ln(x^2+9)$	$dv = 1 dx$
$du = \frac{1}{x^2+9} \cdot (2x) dx$	$v = x$

$$= x \ln(x^2+9) - 2 \int \frac{x^2+9-9}{x^2+9} dx \quad \text{split-split}$$

$$= x \ln(x^2+9) - 2 \left(\int \frac{x^2+9}{x^2+9} - \frac{9}{x^2+9} dx \right)$$

↑ a-rule

$$= x \ln(x^2+9) - 2 \left(x - 9 \cdot \left(\frac{1}{3}\right) \arctan\left(\frac{x}{3}\right) \right) + C$$

OR

$$= x \ln(x^2+9) - 2x + 3 \arctan\left(\frac{x}{3}\right) + C$$