# Mixed Up Series Examples Answer Key

this handout helps figure out Approach

Question: Converge OR// Diverge?

1. $\sum_{n=1}^{\infty} \frac{n^2+n+1}{n^3+n^2+n+1} \approx \sum_{n=1}^{\infty} \frac{n^2}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n}$ Divergent p-Series p=1

LCT Limit

$$\lim_{n\to\infty} \frac{\frac{n^2+n+1}{n^3+n^2+n+1}}{\frac{1}{n}} \; n = \lim_{n\to\infty} \frac{n^3+n^2+n}{n^3+n^2+n+1} \; \frac{\frac{1}{n^3}}{\frac{1}{n^3}}$$

$$= \lim_{n\to\infty} \frac{1+\cancelto{0}{\frac{1}{n}}+\cancelto{0}{\frac{1}{n^2}}}{1+\cancelto{0}{\frac{1}{n}}+\cancelto{0}{\frac{1}{n^2}}+\cancelto{0}{\frac{1}{n^3}}} = \frac{1}{1} = 1$$

Finite
Non-Zero

$\Rightarrow$ Original Series also $\boxed{\text{Diverges by the Limit Comparison Test}}$

2. $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^8+5} \approx \sum_{n=1}^{\infty} \frac{1}{n^8}$ Convergent p-Series p= 8>1

Bound Terms

$$\frac{\sin^2 n}{n^8+5} \le \frac{1}{n^8+5} \le \frac{1}{n^8}$$

$\Rightarrow$ $\boxed{\text{Original Series also Converges by the Comparison Test}}$

Note: LCT not helpful here

3. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^8} \xrightarrow{A.S.} \sum_{n=1}^{\infty} \frac{1}{n^8}$ the Absolute Series Convergent p-Series p= 8>1

Option 1 : ACT

$\Rightarrow$ $\boxed{\text{Original Series Converges by the Absolute Convergence Test}}$

OR// Option 2: AST

1. $b_n = \frac{1}{n^8} > 0$

2. $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \cancelto{0}{\frac{1}{\cancelto{\infty}{n^8}}} = 0$

3. Terms Decreasing

$b_{n+1} = \frac{1}{(n+1)^8} \le \frac{1}{n^8} = b_n$

$\Bigg\} \;\boxed{\text{Series Converges by the Alternating Series Test}}$

4. $\sum_{n=1}^{\infty} \frac{e}{n^e} + \frac{1}{e^n} = \overset{\text{split}}{\sum_{n=1}^{\infty}} \frac{\textcircled{e}}{n^e} + \sum_{n=1}^{\infty} \frac{1}{e^n}$

$$= e \sum_{n=1}^{\infty} \frac{1}{n^e} + \sum_{n=1}^{\infty} \frac{1}{e^n}$$

Constant Multiple
of a Convergent
p-series $p = e > 1$
is Convergent

make Conclusion

+

Convergent by G.S.T. b/c
$|r| = \frac{1}{e} < 1$

Finally, the Sum of two Convergent Series is Convergent

Note: We don't find the Sum since we can't find the Sum of the p-Series even though we can find the Geometric Series Sum.

5. $\sum_{n=1}^{\infty} \frac{n^2}{n^4+5} \approx \sum_{n=1}^{\infty} \frac{n^2}{n^4} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ Convergent p-Series $p = 2 > 1$

<u>Bound Terms</u>

$$\frac{n^2}{n^4+5} \leq \frac{n^2}{n^4} \leq \frac{1}{n^2}$$

$\Rightarrow$ Original Series also Converges by the Comparison Test

Note: Limit Comparison Test also works

6. $\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{5^{n+1}} = -\frac{4}{5^2} + \frac{4^2}{5^3} - \frac{4^3}{5^4} + \ldots$

$a = \frac{-4}{5^2} = \frac{-4}{25}$

$r = \frac{-4}{5}$

Converges by the Geometric Series Test

because $|r| = \left|-\frac{4}{5}\right| = \frac{4}{5} < 1$

Note: No Sum asked here, but the formula is $\text{Sum} = \frac{a}{1-r}$

7. $\sum_{n=1}^{\infty} n^4+5$

$\boxed{\text{Diverges by } n^{th} \text{ Term Divergence Test}}$

because $\lim_{n\to\infty} n^4+5 = \infty \neq 0$

8 $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+1}$

1. $b_n = \frac{1}{\sqrt{n}+1} > 0$

2. $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\cancelto{\infty}{\sqrt{n}+1}} = 0$

3. Terms Decreasing

$b_{n+1} = \frac{1}{\sqrt{n+1}+1} \leq \frac{1}{\sqrt{n}+1} = b_n$

$\boxed{\text{Series Converges by the Alternating Series Test}}$

Note: Absolute Convergence Test isn't helpful since the Absolute Series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$ will Diverge by the LCT

9. $\sum_{n=1}^{\infty} \frac{n^4+5}{n^4+2}$ Diverges by the $n^{th}$ Term Divergence Test nTDT

because

$$\lim_{n\to\infty} \frac{n^4+5}{n^4+2} \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{1+\cancelto{0}{\frac{5}{n^4}}}{1+\cancelto{0}{\frac{2}{n^4}}} = \frac{1}{1} = 1 \neq 0$$

Note: Limit Comparison Test with $\approx \sum 1$ also works, but not it's a bit longer and you get to the same limit

10. $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n^3+n^2+n+1} \longrightarrow \sum_{n=1}^{\infty} \frac{n+1}{n^3+n^2+n+1} \approx \sum_{n=1}^{\infty} \frac{n}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ Convergent p-Series $p=2>1$

LCT Limit

$$\lim_{n\to\infty} \frac{\frac{n+1}{n^3+n^2+n+1}}{\frac{1}{n^2}} \overset{n^2}{} = \lim_{n\to\infty} \frac{n^3+n}{n^3+n^2+n+1} \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{1+\cancelto{0}{\frac{1}{n^2}}}{1+\cancelto{0}{\frac{1}{n}}+\cancelto{0}{\frac{1}{n^2}}+\cancelto{0}{\frac{1}{n^3}}} = \frac{1}{1} = 1$$ Finite Non-Zero

$\Rightarrow$ the Absolute Series also Converges by the Limit Comparison Test

$\Rightarrow$ $\boxed{\text{the Original Series Converges by the Absolute Convergence Test}}$

Note: Alternating Series Test also works, but it may be messy to show terms decreasing

$$11.\quad \sum_{n=1}^{\infty} \frac{1}{e^{2n}} = \frac{1}{e^2} \overset{\cdot \frac{1}{e^2}}{\curvearrowright} + \frac{1}{e^4} \overset{\cdot \frac{1}{e^2}}{\curvearrowright} + \frac{1}{e^6} + \dots$$

$a = \frac{1}{e^2}$

$r = \frac{1}{e^2}$

$\boxed{\text{Converges by Geometric Series Test}}$ because $|r| = \frac{1}{e^2} < 1$

Note: Ratio Test will work. will get $L = \frac{1}{e^2} < 1$

$$12.\quad \sum_{n=1}^{\infty} \frac{6}{n^6} + \frac{1}{6^n} \overset{\text{split}}{=} \sum_{n=1}^{\infty} \frac{6}{n^6} + \frac{1}{6^n}$$

$$= 6\sum_{n=1}^{\infty} \frac{1}{n^6} + \sum_{n=1}^{\infty} \frac{1}{6^n}$$

Constant Multiple of Convergent p-Series $p = 6 > 1$ is Convergent

Convergent Geometric Series $|r| = \frac{1}{6} < 1$

$\boxed{\text{Sum of Two Convergent Series is Convergent}}$

$$13.\quad \sum_{n=1}^{\infty} \frac{\overset{\text{constant}}{\ln 9}}{n^9} = \ln 9 \sum_{n=1}^{\infty} \frac{1}{n^9}$$

Constant Multiple of a Convergent p-Series $p = 9 > 1$ is $\boxed{\text{Convergent}}$

$$14.\quad \sum_{n=2}^{\infty} \frac{n^9}{\ln n}$$

$\boxed{\text{Diverges by } n^{\text{th}} \text{ Term Divergence Test}}$ because

$$\lim_{n\to\infty} \frac{n^9}{\ln n} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{x^9}{\ln x} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{9x^8}{\frac{1}{x}} = \lim_{x\to\infty} 9x^9 = \infty \neq 0$$

$$15.\quad \sum_{n=1}^{\infty} \frac{\ln n}{n^9}$$

$$\int_1^{\infty} \frac{\ln x}{x^9}\,dx = \lim_{t\to\infty} \int_1^t \overset{\overset{u}{L}}{\ln x} \cdot \overset{\overset{dv}{P}}{x^{-9}}\,dx \underset{\text{IBP}}{=} \lim_{t\to\infty} -\frac{\ln x}{8x^8}\Big|_1^t + \frac{1}{8}\int_1^t x^{-9}\,dx \qquad \frac{x^{-8}}{-8}$$

Improper Type I

$$\boxed{\begin{array}{ll} u = \ln x & dv = x^{-9}\,dx \\ du = \frac{1}{x}\,dx & v = \frac{x^{-8}}{-8} \end{array}}$$

$$= \lim_{t\to\infty} \frac{-\ln x}{8x^8}\Big|_1^t - \frac{1}{64x^8}\Big|_1^t$$

$$= \lim_{t\to\infty} \frac{-\overset{\frac{\infty}{\infty}}{\ln t}}{8t^8} + \frac{\cancel{\ln 1}^{\,0}}{8} - \frac{1}{\underset{\infty}{\cancel{64t^8}}}^{\,0} + \frac{1}{64}$$

Continue ↓

Finite

$$\underset{\text{L'H}}{=\lim_{t\to\infty}} \frac{-\frac{1}{t}\;\overset{\frac{1}{64t^7}}{}}{\underset{1}{64t^7}} + \frac{1}{64} = \lim_{t\to\infty} \cancelto{0}{\frac{-1}{\underset{\infty}{64t^8}}} + \frac{1}{64} = \frac{1}{64}$$ Integral Converges

$\Longrightarrow$ $\boxed{\text{Series Converges by the Integral Test}}$

Note: The Improper Integral is not equal to the Series Value, they only share the same Convergence behavior

16. $\sum_{n=1}^{\infty} n^3+n^2+n+1$ $\boxed{\text{Diverges by the } n^{th} \text{ Term Divergence Test}}$

because $\lim_{n\to\infty} \cancelto{\infty}{n^3+n^2+n+1} = \infty \neq 0$

17. $\sum_{n=1}^{\infty} \frac{1}{n!}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{1}{(n+1)!}}{\frac{1}{n!}}\right|^{\nearrow n!} = \lim_{n\to\infty}\frac{\cancel{n!}}{\underset{(n+1)\,\cancel{n!}}{(n+1)!}} = \lim_{n\to\infty}\cancelto{0}{\frac{1}{\underset{\infty}{n+1}}} = 0 < 1$$ (Absolutely) Converges by the Ratio Test.

Note: Since the original series is already the Absolute Series then Absolute Convergence is the Same as $\boxed{\text{Convergence}}$ here.

18. $\sum_{n=1}^{\infty}\left(1+\frac{5}{n^2}\right)^{n^2}$ $\boxed{\text{Diverges by nTDT}}$ because

$1^{\infty}$ $\quad\infty\cdot 0\quad$ flip

$$\lim_{n\to\infty}\left(1+\frac{5}{n^2}\right)^{n^2} = e^{\lim_{n\to\infty}\ln\left(\left(1+\frac{5}{n^2}\right)^{n^2}\right)} = e^{\lim_{x\to\infty} x^2\ln\left(1+\frac{5}{x^2}\right)}$$

$$= e^{\lim_{x\to\infty}\frac{\ln\left(1+\frac{5}{x^2}\right)}{\frac{1}{x^2}}}\ \tfrac{0}{0} \underset{\text{L'H}}{=} e^{\lim_{x\to\infty}\frac{\cancelto{1}{\frac{1}{1+\cancelto{0}{\frac{5}{x^2}}}}\left(\cancelto{5}{\frac{-10}{x^3}}\right)}{\frac{-2}{x^3}}} = e^5 \neq 0$$

$x^{-2} \to -2x^{-3}$

19. $\sum_{n=1}^{\infty}\frac{n^6+3}{n^7} \approx \sum_{n=1}^{\infty}\frac{n^6}{n^7} = \sum_{n=1}^{\infty}\frac{1}{n}$ Divergent (Harmonic) p-Series p=1

Bound Terms

$\frac{n^6+3}{n^7} \geq \frac{n^6}{n^7} = \frac{1}{n}$

$\Rightarrow$ Original Series also $\boxed{\text{Diverges by the Comparison Test}}$

Note: Limit Comparison Test also works here.

20. $\sum_{n=1}^{\infty} 9$ Diverges by nTDT because $\lim_{n\to\infty} 9 = 9 \neq 0$

OR Diverges by GST with $|r| = 1$

$9 + 9 + 9 + \dots$

$\cdot 1$

21. $\sum_{n=1}^{\infty} \frac{(-1)^n}{8^n}$ Converges by GST with $|r| = \left|-\frac{1}{8}\right| = \frac{1}{8} < 1$

OR Ratio Test works too

22. $-1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \dots = -\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots\right) = -\sum_{n=1}^{\infty} \frac{1}{n}$

Constant Multiple of Divergent p-Series $p=1$ is Divergent

23. $\sum_{n=1}^{\infty} \frac{e^n}{\ln n}$ Diverges by nTDT because

$$\lim_{n\to\infty} \frac{e^n}{\ln n} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{e^x}{\ln x} \overset{\infty/\infty}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{e^x}{\frac{1}{x}} = \lim_{x\to\infty} \overset{\infty}{\cancel{x e^x}} = \infty \neq 0$$

Note: Ratio Test also works here. Will get $L = e > 1 \Rightarrow$ Diverges

24. $\sum_{n=1}^{\infty} \arctan\left(\frac{n^7+1}{n^7+7}\right)$ Diverges by nTDT because

$$\lim_{n\to\infty} \arctan\left(\frac{n^7+1}{n^7+7} \frac{\frac{1}{n^7}}{\frac{1}{n^7}}\right) = \lim_{n\to\infty} \arctan\left(\cancel{\frac{1+\overset{0}{\cancel{\frac{1}{n^7}}}}{1+\underset{0}{\cancel{\frac{7}{n^7}}}}}^{\,1}\right) = \frac{\pi}{4} \neq 0$$

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Question: Absolutely Converge, Conditionally Converge OR Diverge?

A. $\sum_{n=1}^{\infty} (-1)^n \frac{n^3+9}{n^9+3} \xrightarrow{A.S} \sum_{n=1}^{\infty} \frac{n^3+9}{n^9+3} \approx \sum_{n=1}^{\infty} \frac{n^3}{n^9} = \sum_{n=1}^{\infty} \frac{1}{n^6}$ Convergent p-Series $p = 6 > 1$

Note: never have to study the original Series

$$\lim_{n\to\infty} \frac{\frac{n^3+9}{n^9+3}}{\frac{1}{n^6}} = \lim_{n\to\infty} \frac{n^9 + 9n^6}{n^9+3} \frac{\frac{1}{n^9}}{\frac{1}{n^9}} = \lim_{n\to\infty} \frac{1 + \overset{0}{\cancel{\frac{9}{n^3}}}}{1 + \overset{0}{\cancel{\frac{3}{n^9}}}} = 1$$

Finite Non-Zero

$\Rightarrow$ Absolute Series also Converges by Limit Comparison Test

$\Rightarrow$ Original Series is Absolutely Convergent by Definition

Recall: to determine if a series is Absolutely Convergent, we must check if the Absolute Series is Convergent, by definition

B. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3n)! \ln n}{(n!)^2 e^{4n} n^n n^6}$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\dfrac{\cancel{(-1)^{n+1}}\,(3(n+1))!\,\ln(n+1)}{((n+1)!)^2\, e^{4(n+1)}\,(n+1)^{n+1}\,(n+1)^6}}{\dfrac{\cancel{(-1)^n}\,(3(n+1))!\,\ln n}{(n!)^2\, e^{4n}\, n^n\, n^6}}\right|$$

$\boxed{(3n+3)(3n+2)(3n+1)}\cancel{(3n)!}$ See ☆ $\quad \to 1$ $\quad \left(\dfrac{n\,\frac{1}{n}}{n+1\,\frac{1}{n}}\right)^6$

$$= \lim_{n\to\infty} \cancel{\frac{(3n+3)!}{(3n)!}} \cdot \cancel{\frac{\ln(n+1)}{\ln n}} \cdot \frac{\cancel{(n!)^2}}{((n+1)!)^2} \cdot \frac{\cancel{e^{4n}}}{e^{4n+4}} \cdot \frac{n^n}{(n+1)^{n+1}} \cdot \frac{n^6}{(n+1)^6}$$

$\boxed{(n+1)^2}\cancel{(n!)^2} \quad \cancel{e^{4n}}\boxed{e^4} \quad (n+1)^n\boxed{(n+1)}$

$\textcircled{3}\cancel{(n+1)}$ $\qquad \frac{1}{e} \qquad 1 \qquad 1 \qquad 0$

$$= \lim_{n\to\infty} \left(\frac{3n+3}{\cancel{n+1}}\right)\left(\frac{3n+2}{n+1}\,\frac{\frac{1}{n}}{\frac{1}{n}}\right)\left(\frac{3n+1}{n+1}\,\frac{\frac{1}{n}}{\frac{1}{n}}\right)\cdot\frac{1}{e^4}\cdot\cancel{\left(\frac{n^n}{(n+1)^n}\right)}\cancel{\left(\frac{1}{1+\frac{1}{n}}\right)^6}$$

$3 \quad 0 \quad 3 \quad 0 \quad 0 \quad 0$

$$= \lim_{n\to\infty} \frac{3}{e^5}\cdot\cancel{\left(\frac{3+\frac{2}{n}}{1+\frac{1}{n}}\right)}\cancel{\left(\frac{3+\frac{1}{n}}{1+\frac{1}{n}}\right)} = \frac{27}{e^5} < 1$$

Converges Absolutely by the Ratio Test

$e \approx 2.7 \Rightarrow e^5 > 32$

☆ $$\lim_{n\to\infty}\frac{\ln(n+1)}{\ln n}^{\frac{\infty}{\infty}} = \lim_{x\to\infty}\frac{\ln(x+1)}{\ln x}^{\frac{\infty}{\infty}} \underset{L'H}{=} \lim_{x\to\infty}\frac{\frac{1}{x+1}}{\frac{1}{x}} = \lim_{x\to\infty}\frac{x}{x+1}^{\frac{\infty}{\infty}} \underset{L'H}{=} \lim_{x\to\infty}\frac{1}{1} = 1$$

C. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(\frac{n+1}{n^2}\right) \xrightarrow{A.S.} \sum_{n=1}^{\infty}\frac{n+1}{n^2} \approx \sum_{n=1}^{\infty}\frac{n}{n^2} = \sum_{n=1}^{\infty}\frac{1}{n}$ Diverges p-Series (Harmonic) $p=1$

1 $b_n = \dfrac{n+1}{n^2} > 0$

Bound Terms

$$\frac{n+1}{n^2} \geq \frac{n}{n^2} = \frac{1}{n}$$

2. $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{n+1}{n^2}\,\frac{1/n^2}{1/n^2}$

$$= \lim_{n\to\infty}\frac{\frac{1}{n}+\cancel{\frac{1}{n^2}}\xrightarrow{}0}{1} = 0$$

or split

Absolute Series also Diverges by Comparison Test

3. Terms Decreasing

$f(x) = \dfrac{x+1}{x^2}$ has

$$f'(x) = \frac{x^2(1)-(x+1)(2x)}{x^4}$$

$$= \frac{x^2-2x^2-2x}{x^4}$$

$$= \frac{\boxed{-}x^2\boxed{-}2x}{x^4} < 0$$

Original Series Converges by Alternating Series Test

Finally, Original Series is **Conditionally Convergent** by Definition

D. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(2n+1)!\,n^n\cdot\pi^n}{(n!)^3\;2^{3n}}$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\dfrac{\cancel{(-1)^{n+1}}(2(n+1)+1)!\,(n+1)^{n+1}\,\pi^{n+1}}{((n+1)!)^3\;2^{3(n+1)}}}{\dfrac{\cancel{(-1)^n}(2n+1)!\,n^n\,\pi^n}{(n!)^3\;2^{3n}}}\right|$$ repartner

$\boxed{(2n+3)(2n+2)}\cancel{(2n+1)!}$ $\quad(n+1)^n\cdot\boxed{(n+1)}$ $\quad\cancel{\pi^n}\cdot\boxed{\pi}$

$$= \lim_{n\to\infty}\frac{(2n+3)!}{\cancel{(2n+1)!}}\cdot\frac{(n+1)^{n+1}}{n^n}\;\frac{\pi^{n+1}}{\cancel{\pi^n}}\;\frac{\cancel{(n!)^3}}{((n+1)!)^3}\cdot\frac{\cancel{2^{3n}}}{2^{3n+3}}$$

$\boxed{2}\cancel{(n+1)}$ $\quad\boxed{(n+1)^3}\cancel{(n!)^3}$ $\quad\cancel{2^{3n}}\cdot\boxed{2^3}$

$$= \lim_{n\to\infty}\left(\frac{2n+3}{n+1}\,\frac{\frac{1}{n}}{\frac{1}{n}}\right)\left(\frac{2n+2}{\cancel{n+1}}\right)\left(\cancel{\frac{n+1}{n+1}}\right)\cdot\pi\cdot\cancel{\frac{(n+1)^n}{n^n}}\xrightarrow{}e\cdot\frac{1}{8}$$

$\pi \approx 3.14$

$e \approx 2.71$

$$= \lim_{n\to\infty}\frac{2\pi}{8}\cdot e\cdot\left(\frac{\cancel{2}\xrightarrow{}2+\cancel{\frac{3}{n}}\xrightarrow{}0}{1+\cancel{\frac{1}{n}}\xrightarrow{}0}\right) = \frac{4\pi e}{8} = \frac{\pi e}{2} > 1$$

**Diverges by Ratio Test**

E. $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n^4+3n+8}{n^9+5}\right) \xrightarrow{A.S.} \sum_{n=1}^{\infty}\frac{n^4+3n+8}{n^9+5} \approx \sum_{n=1}^{\infty}\frac{n^4}{n^9} = \sum_{n=1}^{\infty}\frac{1}{n^5}$ Converges p-Series $p = 5 > 1$

LCT limit

$$\lim_{n\to\infty}\frac{\frac{n^4+3n+8}{n^9+5}}{\frac{1}{n^5}}\curvearrowright n^5 = \lim_{n\to\infty}\frac{n^9+3n^6+8n^5}{n^9+5}\,\frac{\frac{1}{n^9}}{\frac{1}{n^9}} = \lim_{n\to\infty}\frac{1+\cancelto{0}{\frac{3}{n^3}}+\cancelto{0}{\frac{8}{n^4}}}{1+\cancelto{0}{\frac{5}{n^9}}} = 1$$

Finite
Non-Zero

Note: never have to study the original Series

$\Rightarrow$ Absolute Series also Converges by Limit Comparison Test

$\Rightarrow$ original Series is Absolutely Convergent by Definition

F. $\sum_{n=1}^{\infty}\frac{(-1)^n}{8n+3} \xrightarrow{A.S.} \sum_{n=1}^{\infty}\frac{1}{8n+3} \approx \sum_{n=1}^{\infty}\frac{1}{n}$ Diverges p-Series $p=1$

1. $b_n = \frac{1}{8n+3} > 0$

2. $\lim_{n\to\infty} b_n = \lim_{n\to\infty}\cancelto{0}{\frac{1}{8n+3}} = 0$
   $\infty$ show why?

3. Terms Decreasing ← label
   $b_{n+1} = \frac{1}{3(n+1)+8} = \frac{1}{3n+11} \leq \frac{1}{3n+8} = b_n$

label all clearly

Original Series Converges by Alternating Series Test

LCT limit

$$\lim_{n\to\infty}\frac{\frac{1}{8n+3}}{\frac{1}{n}}\curvearrowright n = \lim_{n\to\infty}\frac{n}{8n+3}\,\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \lim_{n\to\infty}\frac{1}{8+\cancelto{0}{\frac{3}{n}}} = \frac{1}{8}$$

Finite
Non-Zero

$\Rightarrow$ Absolute series also Diverges by Limit Comparison Test

Finally, Original Series Conditionally Convergent by Definition

Note: A Series being labelled Conditionally Convergent requires two details

1. Absolute Series is Divergent but/and
2. Original Series is Convergent.