Math 121 Final Exam May 17-26, 2021

Due Wednesday, May 26, in Gradescope by 11:59 pm ET

- This is an *Open Notes* Exam. You can use materials, homework problems, lecture notes, etc. that you manually worked on.
- There is **NO** *Open Internet* allowed. You can only access our Main Course Webpage.
- You are not allowed to work on or discuss these problems with **anyone**. You can only ask me a few small, clarifying, questions about instructions in Office Hours.
- Submit your final work in Gradescope in the **Final Exam** entry.
- Please *show* all of your work and *justify* all of your answers.

1. [10 Points] Compute $\displaystyle\lim_{x\to\infty}\left[1-\arcsin\left(\frac{3}{x^4}\right)-\sin\left(\frac{1}{x^4}\right)\right]^{x^4}$

2. [20 Points] Compute each of the following integrals. Simplify.

(a) Show that $\displaystyle\int_0^{\sqrt{3}}\frac{1}{(1+x^2)^{\frac{7}{2}}}\,dx=\boxed{\frac{49\sqrt{3}}{160}}$

(b) $\displaystyle\int x^4\arcsin x\,dx$

(c) Show that $\displaystyle\int_0^{\sqrt{3}}(x+3)\arctan x\,dx=\boxed{\left(\frac{2}{3}+\sqrt{3}\right)\pi-\frac{\sqrt{3}}{2}-\ln 8}$

3. [40 Points] For each of the following **Improper** integrals, determine whether it converges or diverges. If it converges, find its value. Simplify.

(a) $\displaystyle\int_{-\infty}^{-1}\frac{1}{x^2-6x+25}\,dx$ (b) $\displaystyle\int_{-1}^{6}\frac{15-x}{x^2-6x-7}\,dx$ (c) $\displaystyle\int_0^{e}\frac{\ln x}{\sqrt{x}}\,dx$ (d) $\displaystyle\int_0^{\frac{1}{2}}\frac{1}{x\ln x}\,dx$

4. [28 Points] Determine whether the given series is **Absolutely Convergent, Conditionally Convergent**, or **Divergent**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty}\cos^2\left(\frac{\pi n^6+2021}{6n^6+1}\right)$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\cos^2\left(\pi n^6+2021\right)}{6n^6+1}$

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{\ln(2021)}{n^6}$ (d) $\displaystyle\sum_{n=1}^{\infty}\frac{n^6}{\ln(n+2021)}$

5. [28 Points] Find the **sum** of each of the following series (which do converge). Simplify.

(a) $\frac{5}{3} - 1 + \frac{5}{7} - \frac{5}{9} + \frac{5}{11} - \ldots$ (b) $\frac{1}{2} - \frac{1}{8} + \frac{1}{3 \cdot 2^3} - \frac{1}{64} + \frac{1}{5 \cdot 2^5} - \ldots$ (c) $\sum_{n=0}^{\infty} \frac{(-3)^n - 2}{4^n}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n \, \pi^{2n+1}}{(\sqrt{2})^{4n} \, (2n)!}$ (e) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} \, (2^4) \, \pi^{2n}}{2^{4n} \, (2n)!}$

(f) $-\pi + \frac{\pi^3}{3!} - \frac{\pi^5}{5!} + \frac{\pi^7}{7!} - \frac{\pi^9}{9!} + \ldots$ (g) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} \, (\ln 9)^n}{2^{n+1} \, n!}$

6. [21 Points] Find the **Interval** and **Radius** of Convergence for the following Power Series.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n \, (n+1) \, (5x-2)^n}{n^2 \, 8^n}$ (b) $\sum_{n=1}^{\infty} \frac{n^n \, (\ln n) \, (x-7)^n}{(2n)! \, e^n \, \sqrt{n}}$

7. [10 Points] Use MacLaurin Series to **Estimate** $\ln\left(\frac{3}{2}\right)$ with error less than $\frac{1}{50}$. Simplify.

8. [12 Points]

(a) Find the MacLaurin Series for $F(x) = \ln(x+3)$ and **State** the Radius of Convergence. Your answer should be in Sigma notation $\sum_{n=0}^{\infty}$ here, except for one term.

- **Use** the Fact that: $\ln(3+x) = \int \frac{1}{x+3} \, dx$

(b) Use a **Different** Method from part (a) to find the MacLaurin Series for $F(x) = \ln(x+3)$.

- **Use** the Algebra Fact that: $\ln(x+3) = \ln\left[3\left(1+\frac{x}{3}\right)\right] = \ln 3 + \ln\left(1+\frac{x}{3}\right)$

**Check that your Answers in (a) and (b) match up. For your own fun, you can use the Definition of the MacLaurin Series (Chart Method) to confirm your answer.

9. [10 Points] **Compute** the Area bounded by the Polar Cuve $r = 2 - 2\sin\theta$. Sketch the curve.

10. [21 Points] For **each** of the following problems, do the following **THREE** things:

1. **Sketch** the Polar curve(s) and **shade** the described bounded region.

2. Set-Up but **DO NOT EVALUATE** an Integral representing the area of the described bounded region.

3. Set-Up but **DO NOT EVALUATE** another **slightly different** Integral representing the same area of the described bounded region.

(a) The **area** bounded outside the polar curve $r = 1$ and inside $r = 2\sin\theta$.

(b) The **area** that lies inside both of the curves $r = 2 + 2\sin\theta$ and $r = 2 - 2\sin\theta$.

(c) The **area** bounded outside the polar curve $r = 1 + \cos\theta$ and inside $r = 3\cos\theta$.

(d) The **area** bounded inside the polar curve $r = 3 + 2\cos\theta$.

- You will need to use the Cartesian Plot to help you sketch this Limaçon Cardioid.
- No worries, just try this new polar curve. You will be graded on effort only here.

Have a Wondeful and Relaxing Summer!

Come visit me at my office next year for candy and chocolate!!