Math 111, Section 01, Fall 2012

Worksheet 11, Thursday, December 6, 2012

1. Consider $f(x) = \dfrac{e^x}{e^x + 1}$

   (a) Compute $f'(x)$ using the Quotient Rule.

   (b) Use the Limit Definition of the Derivative to compute $f'(x)$. Check that your answers are equal. You might need to use a special limit fact about the natural exponential. Talk to your partner about *how* we met the Natural Exponential function.

2. Follow the next few steps to prove that the derivative

$$\boxed{\frac{d}{dx} \ln x = \frac{1}{x}}.$$

   (a) Start with $y = \ln x$.

   (b) Exponentiate both sides so that $e^y = x$.

   (c) Implicitly differentiate both sides with respect to x.

   (d) Solve for $\dfrac{dy}{dx}$ in terms of y

   (e) Resubsitute $y = \ln x$ to solve for $\dfrac{dy}{dx}$ in terms of x.

3. Compute $\displaystyle\int \tan x \, dx$.

4. Compute $\displaystyle\int_0^{\ln 2} \frac{e^{3x}}{\sqrt{8 + e^{3x}}} \, dx$.

5. Compute $\displaystyle\int_0^1 \frac{e^x}{2 + e^x} \, dx$.

6. Compute $\displaystyle\int_{e^3}^{e^9} \frac{1}{5x} \, dx$.

7. Compute $\displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}} \, dx$.

8. Compute $\displaystyle\int_e^{e^4} \frac{3}{x\sqrt{\ln x}} \, dx$.

9. Compute $\int \frac{\sqrt{1+e^{-x}}}{e^x}\, dx$.

10. Compute $\int \frac{(\ln x)^2}{x}\, dx$

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Recall from class that the formula for Volumes of Revolution using the **Disk Method** and rotating about the x-axis was:

$$V = \int_a^b \pi\ (\text{radius})^2\ dx$$

11. Let R be the region bounded by $y = e^x$, the x-axis, $x = 0$, and $x = 2$. Compute the volume of the solid formed by rotating R about the x-axis. Sketch the solid as well as one of the approximating disks.

    Note: You should sketch both the 2 and 3-dimensional sketches.

    Hint: To sketch one of the approximating disks, first sketch the approximating rectangle (from Area-Riemann sums days) in the 2-dimensional sketch. Then think about how that approximating rectangle spins around the axis.

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Recall from class that the formula for Volumes of Revolution using the **Washer Method** and rotating about the x-axis was:

$$V = \int_a^b \pi\ \left[(\text{outer radius})^2 - (\text{inner radius})^2\right]\ dx$$

12. Let R be the region bounded by $y = e^x + 1$, $y = x + 1$, $x = 0$, and $x = 1$. Compute the volume of the solid formed by rotating R about the x-axis. Sketch the solid as well as one of the approximating washers.

Turn in your own solutions.