Worksheet # 10

1. $g(x) = \int_{\tan x}^{7} \sqrt{e^t + 3}\, dt$

$$g'(x) = \frac{d}{dx}\int_{\tan x}^{7} \sqrt{e^t+3}\, dt = -\frac{d}{dx}\int_{7}^{\tan x} \sqrt{e^t+3}\, dt$$

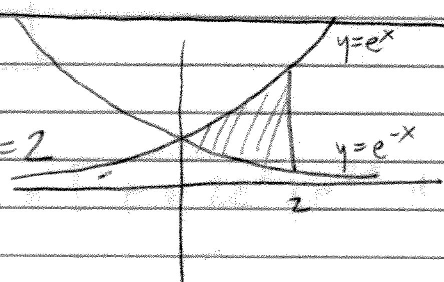
$$= -\sqrt{e^{\tan x}+3}\cdot \sec^2 x$$

$$g''(x) = \boxed{-\sqrt{e^{\tan x}+3}\cdot 2\sec x\cdot \sec x\tan x \mp \sec^2 x\cdot \frac{1}{2\sqrt{e^{\tan x}+3}}\cdot e^{\tan x}\cdot \sec^2 x}$$

2. $f(x) = \sqrt{\cos(x^2+e^x)} + \cos\sqrt{x^2+e^x} + e^{\sqrt{x^2+\cos x}}$

$$f'(x) = \boxed{\frac{1}{2\sqrt{\cos(x^2+e^x)}}\cdot\left(-\sin(x^2+e^x)\right)(2x+e^x) - \sin\sqrt{x^2+e^x}\cdot\frac{1}{2\sqrt{x^2+e^x}}(2x+e^x) + e^{\sqrt{x^2+\cos x}}\frac{1}{2\sqrt{x^2+\cos x}}(2x-\sin x)}$$

3. $y=e^x,\ y=e^{-x},\ y=2$



$$\text{Area Bounded} = \int_0^2 e^x - e^{-x}\,dx = e^x + e^{-x}\Big|_0^2 = (e^2+e^{-2}) - (\cancel{e^0}^{\,1} + \cancel{e^0}^{\,1})$$

$$= \boxed{e^2 + \frac{1}{e^2} - 2}$$

4. (a) $\displaystyle\int_{\pi/6}^{\pi/3} \sec^2 x \tan^3 x \, dx = \int_{1/\sqrt{3}}^{\sqrt{3}} u^3 \, du = \frac{u^4}{4}\Bigg|_{1/\sqrt{3}}^{\sqrt{3}} = \frac{(\sqrt{3})^4}{4} - \frac{\left(\frac{1}{\sqrt{3}}\right)^4}{4}$

$\boxed{\begin{array}{l} u = \tan x \\ du = \sec^2 x\,dx \end{array}}$ $\boxed{\begin{array}{l} x = \pi/6 \Rightarrow u = \tan \pi/6 = \frac{1}{\sqrt{3}} \\ x = \pi/3 \Rightarrow u = \tan \pi/3 = \sqrt{3} \end{array}}$

$= \frac{9}{4} - \frac{1}{36} = \frac{81}{36} - \frac{1}{36} = \frac{80}{36}$

$= \boxed{\frac{20}{9}}$

(b) $\displaystyle\int \frac{1}{\sqrt{x}(1+\sqrt{x})^9} \, dx = 2\int \frac{1}{u^9} \, du = 2\frac{u^{-8}}{-8} + C = -\frac{1}{4}\frac{1}{u^8} + C = \boxed{\frac{-1}{4(1+\sqrt{x})^8} + C}$

u^{-9}

$\boxed{\begin{array}{l} u = 1+\sqrt{x} \\ du = \frac{1}{2\sqrt{x}}dx \\ 2du = \frac{1}{\sqrt{x}}dx \end{array}}$

(c) $\displaystyle\int x\sqrt{x+1} \, dx = \int (u-1)\sqrt{u} \, du = \int u^{3/2} - u^{1/2} \, du = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$\boxed{\begin{array}{l} u = x+1 \Rightarrow x = u-1 \\ du = dx \end{array}}$ inverted substitution $= \boxed{\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C}$

(d) $\displaystyle\int \frac{(1+e^x)^2}{e^x} \, dx = \int \frac{1+2e^x+e^{2x}}{e^x} \, dx = \int \frac{1}{e^x} + 2 + e^x \, dx$

e^{-x}

$= \boxed{-e^{-x} + 2x + e^x + C}$

(e) $\displaystyle\int_1^9 \sqrt{2x+7} \, dx = \frac{1}{2}\int_9^{25} \sqrt{u}\,du = \frac{1}{\cancel{2}}\frac{\cancel{2}}{3}u^{3/2}\Bigg|_9^{25} = \frac{1}{3}(\sqrt{25})^3 - \frac{1}{3}(\sqrt{9})^3$

$u^{1/2}$

$\boxed{\begin{array}{l} u = 2x+7 \\ du = 2dx \\ \frac{1}{2}du = dx \end{array}}$ $\boxed{\begin{array}{l} x = 1 \Rightarrow u = 9 \\ x = 9 \Rightarrow u = 25 \end{array}}$

$= \frac{1}{3}(125) - \frac{1}{3}(27)$

$= \frac{125}{3} - \frac{27}{3} = \boxed{\frac{98}{3}}$

4(f). $\int (e^{x}+e^{-x})^2\,dx = \int e^{2x}+2+e^{-2x}\,dx = \boxed{\frac{1}{2}e^{2x}+2x-\frac{1}{2}e^{-2x}+c}$

5. $f(x) = \int f'(x)\,dx = \int x^2\sin(x^3)\,dx = \frac{1}{3}\int \sin u\,du = -\frac{1}{3}\cos u + C = -\frac{1}{3}\cos(x^3)+C$

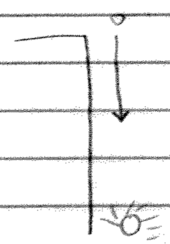
$$\boxed{\begin{array}{l} u=x^3 \\ du=3x^2dx \\ \frac{1}{3}du=x^2dx \end{array}}$$

Use Initial Condition $f(0)=3$.

$f(0) = -\frac{1}{3}\underbrace{\cos(0)}_{1} + C \overset{set}{=} 3$

$\Rightarrow \quad C = 3+\frac{1}{3} = \frac{10}{3}$

Finally, $\boxed{f(x) = -\frac{1}{3}\cos(x^3)+\frac{10}{3}}$

6. 

$a(t) = -32$ ft/sec^2

$v(0) = -96$ ft./sec

$s(0) = ?$

$v(t) = -32t + v(0)$

$= -32t - 96$

looking for

$s(t) = -16t^2 - 96t + s(0)$

$s(1) = \underbrace{-16 - 96}_{} + s(0) = 0$ ← hits ground

$-112 + s(0) = 0$

$\Rightarrow s(0) = 112$

Finally, height of building (which is same as initial position of the ball) is 112 feet

7. FTC

$$\int_2^5 x^2\,dx = \frac{x^3}{3}\Big|_2^5 = \frac{125}{3} - \frac{8}{3} = \frac{117}{3} = \boxed{39}$$

Riemann Sums + Limit Definition

$a=2 \quad b=5 \quad \Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n} \qquad x_i = a + i\Delta x = 2 + 3i/n$

$$\int_2^5 x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f(2+3i/n)\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n}\left(2+3i/n\right)^2 = \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 4 + \frac{12i}{n} + \frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\overbrace{\sum_{i=1}^{n}4}^{4n} + \frac{3}{n}\sum_{i=1}^{n} 12i/n + \frac{3}{n}\sum_{i=1}^{n}\frac{9i^2}{n^2}\right)$$

$$= \lim_{n\to\infty} \frac{12}{\cancel{n}}(\cancel{n}) + \frac{36}{n^2}\sum_{i=1}^{n} i + \frac{27}{n^3}\sum_{i=1}^{n} i^2$$

$$= \lim_{n\to\infty} 12 + \frac{36}{n^2}\left[\frac{n(n+1)}{2}\right] + \frac{27}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} 12 + 18\left(\frac{\cancel{n}}{\cancel{n}}\right)\left(\frac{n+1}{n}\right) + \frac{27}{6}\left(\frac{\cancel{n}}{\cancel{n}}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$$

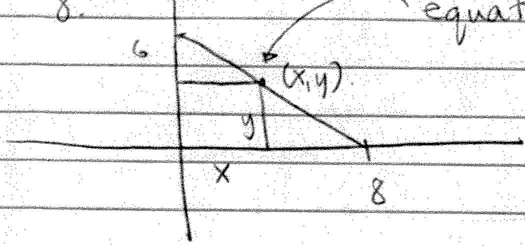
$$= \lim_{n\to\infty} 12 + 18\left(1+\underbrace{\cancel{1/n}}_{0}\right) + \underbrace{\frac{27}{6}\left(1+\overbrace{\cancel{\tfrac{1}{n}}}^{0}\right)\left(2+\overbrace{\cancel{\tfrac{1}{n}}}^{0}\right)}_{\frac{54}{6}}$$

$$= 12 + 18 + 9$$

$$= \boxed{39} \quad \text{Match!}$$

• Diagram

8.



equation of line $y = -\frac{3}{4}x + 6$.

OR

(Similar Triangles)

$\frac{6-y}{6} = \frac{x}{8}$

$48 - 8y = 6x$

$\Rightarrow y = \frac{-6x+48}{8}$

$= \frac{-3x}{4} + 6$

• Variables let x = width of inscribed rectangle
y = height of inscribed rectangle
A = Area of rectangle.

• Equations

$y = -\frac{3}{4}x + 6$ Fixed!

$A = xy$ Maximize

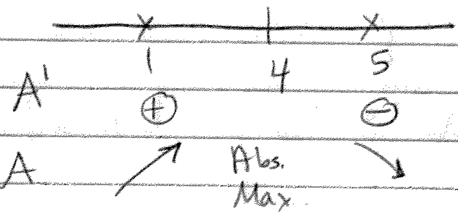
$= x\left(-\frac{3}{4}x + 6\right) = -\frac{3}{4}x^2 + 6x$ Domain $0 \le x \le 8$.

• Maximize

$A' = -\frac{6}{4}x + 6 \overset{set}{=} 0$

$6 = \frac{6}{4}x \Rightarrow x = 4$ critical number.

• Sign Testing



$x = 4 \Rightarrow A(4) = -\frac{3}{4}(4)^2 + 6(4) = -12 + 24 = 12$.

OR $x = 4 \Rightarrow y = -\frac{3}{4}(4) + 6 = -3 + 6 = 3 \Rightarrow A = xy = 4 \cdot 3 = 12$.

• Answer: The Maximum possible Area is 12 square units.