Worksheet #8

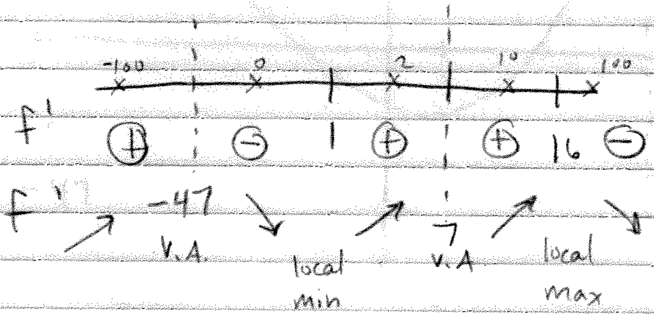
1. $f(x) = \dfrac{x-9}{(x-7)(x+47)^2}$

- Domain $\{x : x \neq 7, -47\}$
- V.A. $x = 7,\ x = -47.$
- H.A. $\lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{x-9}{x^3+87x^2+1551x-15463} \dfrac{(1/x^3)}{(1/x^3)} = \lim\limits_{x\to\pm\infty} \dfrac{\overset{0}{\cancel{1/x^2}} - \overset{0}{\cancel{9/x^3}}}{1+\underset{0}{\cancel{87/x}}+\underset{0}{\cancel{1551/x^2}}-\underset{0}{\cancel{15463/x^3}}} = \dfrac{0}{1} = 0.$

H.A. @ $y=0.$

- First Derivative given $f'(x) = \dfrac{-2(x-1)(x-16)}{(x-7)^2(x+47)^3} \overset{\text{set}}{=} 0$ Critical #'s $x = 1, 16$

- Sign Testing into f'



f Increasing $(-\infty,-47) \cup (1,7) \cup (7,16)$

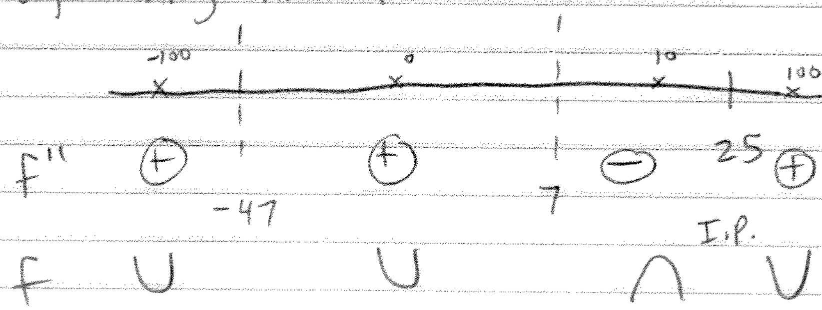
Decreasing $(-47,1) \cup (16,\infty)$

Local Min $(1, f(1)) = ?$

Local Max $(16, f(16)) = ?$

- Second Derivative given $f''(x) = \dfrac{6(x-25)(x^2+59)}{(x-7)^3(x+47)^4}$ $\Rightarrow x = 25$ possible I.P.

- Sign Testing into f''



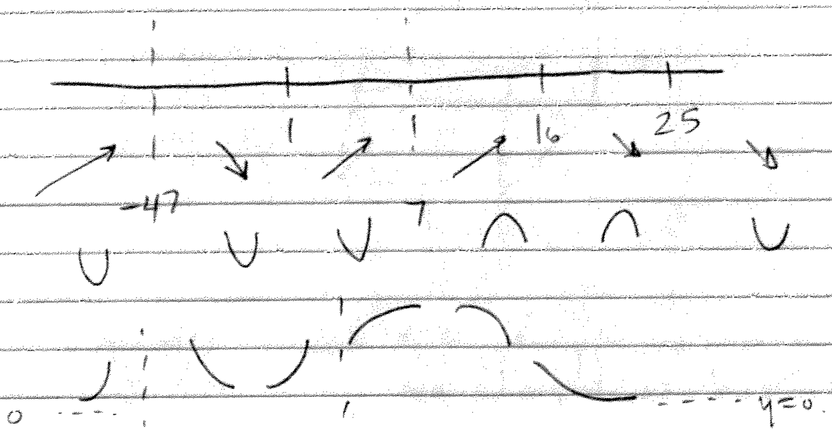
C.U $(-\infty,-47) \cup (-47,7) \cup (25,\infty)$

C.D. $(7,25)$

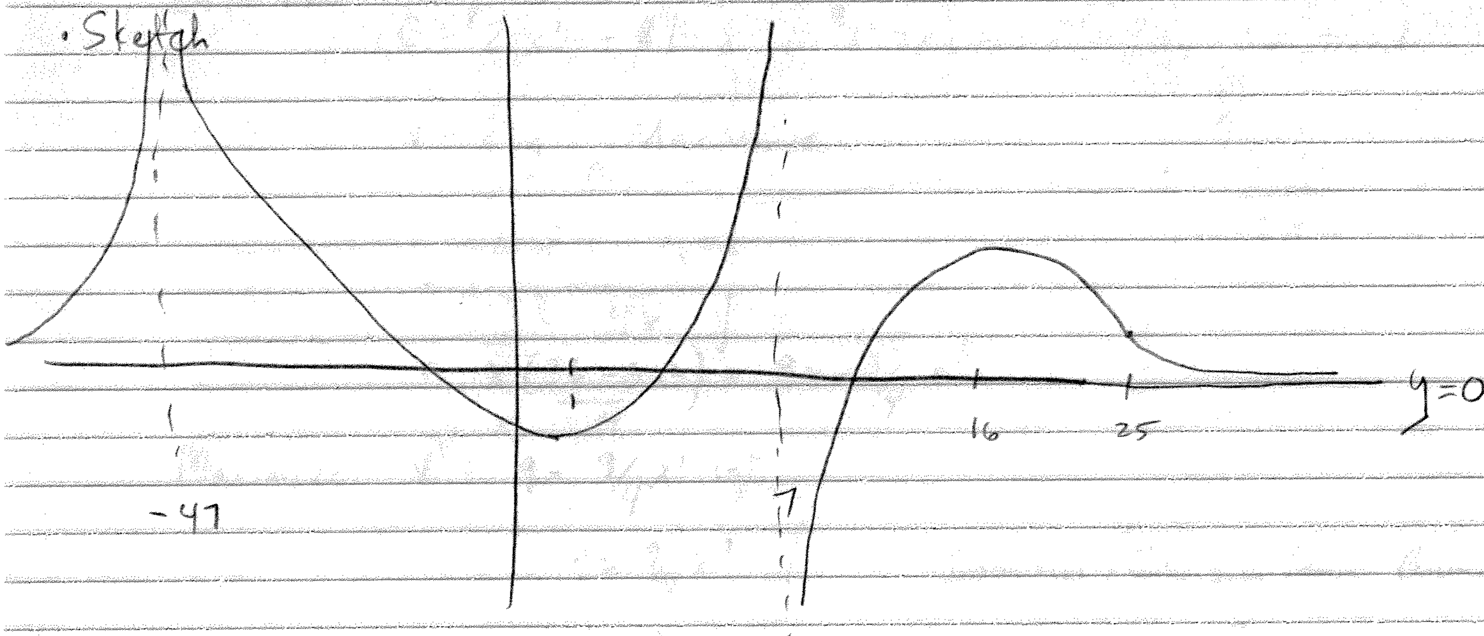
I.P. $(25, f(25)) = ?$

· Piece Together

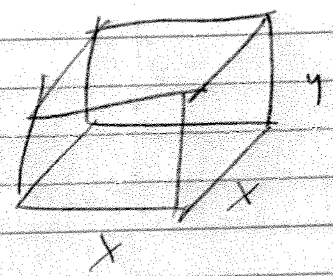
$$f(0) = \frac{-9}{-7(47)^2}$$

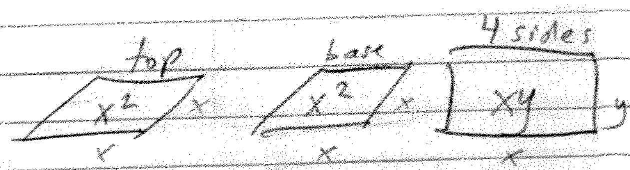


· Sketch



2. · Diagram





· Variables let x = length and width of base of box
y = height of box.

· Equations. $C = \$2x^2 + \$1 \cdot x^2 + \$1 \cdot 4xy = 3x^2 + 4xy = 36$ fixed

$\Downarrow$

$y = \dfrac{36 - 3x^2}{4x}$

$V = x^2 y$ Maximize.

$$= x^2 \left[\frac{36 - 3x^2}{4x}\right]$$

$$= x\left(\frac{36 - 3x^2}{4}\right) = 9x - \tfrac{3}{4}x^3$$

· Maximize: $V' = 9 - \tfrac{9}{4}x^2 \overset{\text{set}}{=} 0.$

$9 = \tfrac{9}{4}x^2$

$\Rightarrow x^2 = 4$

$x = \pm 2$ critical #.
ignore $\ominus$

Domain / Common Sense Bounds.

$\{x : x > 0\}$

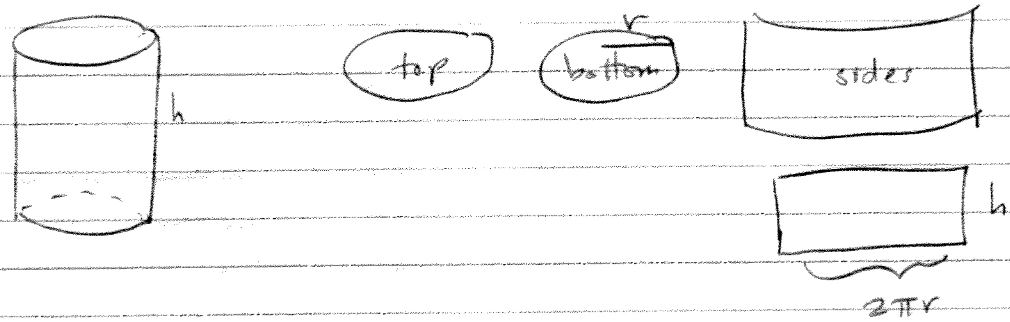
· Sign Testing

	(0	test $x = 1$	2	test $x = 10$
V'		$\oplus$		$\ominus$
V		$\nearrow$	Abs. Max	$\searrow$

$x = 2 \Rightarrow y = \dfrac{36 - 3(4)}{8} = \dfrac{24}{8} = 3$

· Answer The box with largest volume has dimensions 2ft. x 2ft. x 3ft

3. · Diagram



· Variables let r = radius of can

h = height of can

V = Volume of can

M = amount of Material to make Can (Surface Area).

· Equations

$$V = \pi r^2 h = 2000\pi \quad \text{Fixed} \Rightarrow h = \frac{2000}{r^2}$$

$$M = 2\pi r^2 + 2\pi r h \quad \text{Minimize}$$

$$= 2\pi r^2 + 2\pi r\left(\frac{2000}{r^2}\right)$$

$$= 2\pi r^2 + \frac{4000\pi}{r}$$

Domain / Common Sense Bounds.

$\{r : r > 0\}$.

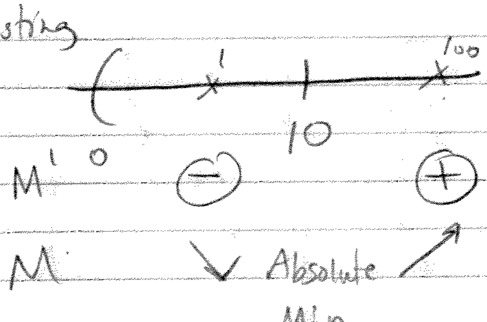
· Minimize: $M' = 4\pi r - \dfrac{4000\pi}{r^2} \overset{\text{set}}{=} 0$

$$4\pi r = \frac{4000\pi}{r^2}$$

$$r^3 = 1000$$

$$r = 10 \quad \text{critical \#}$$

· Sign Testing



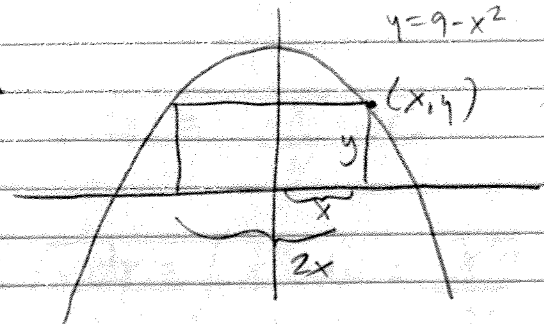
$$r = 10 \Rightarrow h = \frac{2000}{100} = 20.$$

Worksheet #8 (4)

· Answer: The can with least surface area has dimensions of radius 10 in and height 20 inches.

check $V = \pi(10)^2(20) = 2000\pi$ ✓

4. · Diagram



· Variables Let x = x-coordinate of point (x,y) on parabola.

y = y-coordinate " " "

A = Area of inscribed rectangle

· Equations: $y = 9 - x^2$ Fixed! (Given already)

$A = (2x)y$ Maximize!

$= 2x(9-x^2)$

$= 18x - 2x^3$

Common Sense Bounds

$\{x \mid 0 \le x \le 3\} = [0,3]$.

· Maximize

$A' = 18 - 6x^2 \overset{set}{=} 0$

$x^2 = 18/6 = 3$

$x = \pm\sqrt{3}$ critical #

· Sign Testing into A'

	0	1	$\sqrt{3}$	2	3
A'		(+)		(−)	
A		↗	Abs. Max	↘	

$A(\sqrt{3}) = 2\sqrt{3}\left(9 - (\sqrt{3})^2\right)$

$= 2\sqrt{3}(9-3)$

$= 12\sqrt{3}$

· Answer: The largest rectangle inscribed as described has Area $12\sqrt{3}$ square units.

5. a. $\int x^3 dx = \frac{x^4}{4} + C$ $\quad \frac{d}{dx} \frac{x^4}{4} + C = x^3$ ✓

b. $\int \sqrt{x}\, dx = \int x^{1/2} dx = \frac{x^{3/2}}{3/2} + C = \frac{2}{3} x^{3/2} + C$ $\quad \frac{d}{dx} \frac{2}{3} x^{3/2} + C = x^{1/2}$ ✓

c. $\int \cos x\, dx = \sin x + C$ $\quad \frac{d}{dx} \sin x + C = \cos x$ ✓

d. $\int \sin x\, dx = -\cos x + C$ $\quad \frac{d}{dx} -\cos x + C = -(-\sin x) = \sin x$ ✓

e. $\int \sec^2 x\, dx = \tan x + C$ $\quad \frac{d}{dx} \tan x + C = \sec^2 x$ ✓

f. $\int \sec x \tan x\, dx = \sec x + C.$ $\quad \frac{d}{dx} \sec x = \sec x \tan x$

g. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ $\quad \frac{d}{dx} \frac{x^{n+1}}{n+1} + C = x^n$ ✓

h. $f(x) = x^3(1+x^2) = x^3 + x^5$ $\quad$ No Product Rule Here!!

$\int x^3 + x^5 dx = \boxed{\frac{x^4}{4} + \frac{x^6}{6} + C}$ $\quad \frac{d}{dx}\left(\frac{x^4}{4} + \frac{x^6}{6} + C\right) = x^3 + x^5$ ✓

i. $f(x) = \frac{x + \sqrt{x} + 7}{x^3} = \frac{x}{x^3} + \frac{\sqrt{x}}{x^3} + \frac{7}{x^3} = \frac{1}{x^2} + \frac{1}{x^{5/2}} + 7/x^3$

$= x^{-2} + x^{-5/2} + 7x^{-3}$

$\int x^{-2} + x^{-5/2} + 7x^{-3} dx = \frac{x^{-1}}{-1} + \frac{x^{-3/2}}{-3/2} + 7\frac{x^{-2}}{-2} + C$

$= -1/x - \frac{2}{3} x^{-3/2} - 7/2 \frac{1}{x^2} + C.$

$\frac{d}{dx}\left(-x^{-1} - 2/3\, x^{-3/2} - 7/2\, x^{-2} + C\right) = x^{-2} + x^{-5/2} + 7x^{-3}$ ✓