Math 111, Section 01, Fall 2012

Worksheet 8, Thursday, November 1, 2012

1. Let $f(x) = \dfrac{x-9}{(x-7)(x+47)^2} = \dfrac{x-9}{x^3 + 87x^2 + 1551x - 15463}$

   Take my word for it that

   $$f'(x) = \frac{-2(x-1)(x-16)}{(x-7)^2(x+47)^3} \quad \text{and} \quad f''(x) = \frac{6(x-25)(x^2+59)}{(x-7)^3(x+47)^4}$$

   For this function, discuss domain, vertical and horizontal asymptotes, interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present and detailed and labelled sketch of the curve.

2. A rectangluar box with square base. The material costs $2 per square foot for the bottom, and $1 per square foot for the top *and* sides. Find the box of largest volume which can be built for $36.

3. A cylindrical can has a volume of $2000\,\pi$ cubic inches. Determine the height and radius of the can that has the least surface area.

4. Let R be a region enclosed between $y = 9 - x^2$ and the x–axis. Find the area of the largest rectangle that can be inscribed in this region R.

   Note: Two vertices of the inscribed rectangle lie on the x–axis. The other two lie on the parabola $y = 9 - x^2$.

5. Definition: Take F, f functions defined on an interval I and suppose that $F'(x) = f(x)$ on I. Then

- $F(x)$ is called **an** antiderivative of $f(x)$
- $F(x) + C$ is called the **most general** antiderivative of $f(x)$, where C is any constant.

We will use the notation $\int f(x) \, dx$ to denote the most general antiderviative.

For example: $\int x^7 \, dx = \dfrac{x^8}{8} + C$ because the derivative of that most general antiderivative does indeed equal x^7. We have found **all** possible antiderivatives. Note that $\dfrac{x^8}{8} + 3$ is **an** antiderivative of x^7. So is $\dfrac{x^8}{8} + 2012$ as well as $\dfrac{x^8}{8} - 5$.

Hint: if you ever want to know whether you found the correct antiderivative, take the deriative of your answer and check that you return to the original function.

For each of the following differentiate you answer to double check that you have the correct most general antiderivative.

(a) Compute $\int x^3 \, dx$

(b) Compute $\int \sqrt{x} \, dx$

(c) Compute $\int \cos x \, dx$

(d) Compute $\int \sin x \, dx$

(e) Compute $\int \sec^2 x \, dx$

(f) Compute $\int \sec x \tan x \, dx$

(g) Find a general power rule for $\int x^n \, dx$ where n is any real number with $n \neq -1$. (We will learn the $n = -1$ case at the very end of this semester.)

(h) Find the most general antiderivative of $f(x) = x^3(1 + x^2)$.

(i) Find the most general antiderivative of $f(x) = \dfrac{x + \sqrt{x} + 7}{x^3}$.

Turn in your own solutions.