

## Worksheet 4, Thursday, September 27, 2012

- Please *show* all of your work and *justify* all of your answers.

**1.** Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value,  $+\infty$  or  $-\infty$ , or Does Not Exist.

(a) 
$$\lim_{x \rightarrow 7} \frac{x^2 - 4x - 21}{x^2 - 3x} =$$

(b) 
$$\lim_{x \rightarrow 4} \frac{x^2 - 3x - 4}{|x - 4|} =$$

(c) 
$$\lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x^2 - 5x + 4} =$$

(d) 
$$\lim_{x \rightarrow -5} \frac{\frac{1}{1-x} - \frac{1}{6}}{x^2 + 3x - 10} =$$

(e) 
$$\lim_{x \rightarrow 3} \frac{x^2 - 12x + 27}{x^2 - 6x + 9} =$$

(f) 
$$\lim_{x \rightarrow 3} \frac{x^2 - 12x + 27}{x^2 - 9} =$$

(g) 
$$\lim_{x \rightarrow 4} \frac{x + 2}{4 - x} =$$

(h) 
$$\lim_{x \rightarrow -4} \frac{x + 2}{x + 4} =$$

(i) 
$$\lim_{x \rightarrow 2} \frac{3 - \sqrt{x+7}}{x^2 - 3x + 2} =$$

(j) 
$$\lim_{x \rightarrow 1} \frac{G(x+2) + x - 8}{G(2x) - 3x^2 - 3x + 2} = \quad \text{where } G(x) = (x-1)^2 + 3$$

(k) 
$$\lim_{x \rightarrow 7} \frac{x - 7}{|7 - x|} =$$

(l) 
$$\lim_{x \rightarrow 5} \frac{f(x^2) - 28}{(f(x))^2 - 10x - 14} = \quad \text{where } f(x) = x + 3$$

**2.** Prove that  $\lim_{x \rightarrow 3} 1 - 2x = -5$  using the  $\varepsilon - \delta$  definition of the limit.

**3.** (a) Suppose that  $f(x) = \sqrt{x^2 - 5x + 3}$ . Compute  $f'(x)$  using the **limit definition of the derivative**.

(b) Suppose that  $f(x) = \frac{1 - 3x}{x + 2}$ . Compute  $f'(x)$  using the **limit definition of the derivative**.

**4.** Suppose that  $f(x) = 5 - 7x + 4x^2 - x^3$ .

(a) Write the **equation of the tangent line** to the curve  $y = f(x)$  when  $x = 1$ . **\*\*Use the limit definition of the derivative when computing the derivative.\*\***

(b) Find the  $x$ -coordinate(s), if any, where the tangent line to  $f(x)$  is horizontal.

5. State the definition for a function  $f(x)$  that is continuous at  $x = -7$ .

6. Suppose that  $f$  and  $g$  are functions, **and**

- $\lim_{x \rightarrow 7} g(x) = 3$
- $\lim_{x \rightarrow 2} g(x) = 6$
- $f(3) = 2$
- $g(x)$  is continuous at  $x = 7$  and  $x = 2$ .
- $\lim_{x \rightarrow 3} f(x) = 5$ .

Evaluate the following quantities and fully **justify** your answers. Do **not** just put down numbers.

(a)  $g(7) =$

(b)  $g \circ f(3) =$

(c)  $f \circ g(7) =$

(d) Is  $f(x)$  continuous at  $x = 3$ ? Why or why not?

7. Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-3} + 1 & \text{if } x > 3 \\ 0 & \text{if } x = 3 \\ (x-2)^2 & \text{if } 1 < x < 3 \\ x + \frac{1}{3} & \text{if } 0 < x \leq 1 \\ \frac{1}{x+3} & \text{if } x < 0 \end{cases}$$

(a) Carefully sketch the graph of  $f(x)$ .

(b) State the **Domain** of the function  $f(x)$ .

(c) Compute  $\lim_{x \rightarrow -3} f(x) =$

(d) Compute  $\lim_{x \rightarrow 0} f(x) =$

(e) Compute  $\lim_{x \rightarrow 1} f(x) =$

(f) Compute  $\lim_{x \rightarrow 3} f(x) =$

(g) State all the value(s) at which  $f$  is discontinuous. Justify your answer(s) using the definition of continuity.