

**Worksheet 3, Thursday, September 20, 2012**

1. Compute the following limits. Be clear if they equal a value, or  $+\infty$ ,  $-\infty$ , or DNE.

(a)  $\lim_{x \rightarrow 2} \frac{x^2 - 9x + 14}{x^2 - 4x + 4}$

(b)  $\lim_{x \rightarrow 2} \frac{3 - \sqrt{x+1}}{x-8}$

(c)  $\lim_{x \rightarrow 8} \frac{3 - \sqrt{x+1}}{x-8}$

(d)  $\lim_{x \rightarrow 2} \frac{x^2 - 6 + |x - 4|}{3x - 6}$

(e)  $\lim_{x \rightarrow 2} \frac{x-2}{|2-x|}$

2. Write out the rigorous  $\epsilon - \delta$  **Definition of the Limit**  $\lim_{x \rightarrow a} f(x) = L$ .

3. Give an  $\epsilon - \delta$  proof that  $\lim_{x \rightarrow 1} 10 - 7x = 3$ .

4. Give an  $\epsilon - \delta$  proof that  $\lim_{x \rightarrow 6} 4 - \frac{3x}{2} = -5$ .

5. Let  $f(x)$  be a function with the property  $\lim_{x \rightarrow 2} f(x) = 5$ .

(a) Discuss what you can conclude about your function  $f(x)$ .

(b) Discuss what you know about  $f(2)$ . Explain your reasoning.

6. Consider the function  $f(x)$  that is continuous at  $x = 3$ . Assume that  $f(3) = 4$ .

(a) Write the definition for  $f(x)$  being continuous at  $x = 3$ .

(b) Discuss what you know about  $\lim_{x \rightarrow 3} f(x) = ??$ .

$$7. \text{ Let } h(x) = \begin{cases} \frac{8}{x+2} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \frac{1}{2}x - 4 & \text{if } 0 < x < 16 \\ 0 & \text{if } x = 16 \\ \sqrt{x} & \text{if } x > 16 \end{cases}$$

Answer the following questions:

- (a) Sketch the graph of  $h(x)$ . State the Domain of  $h(x)$ .
- (b) Compute  $\lim_{x \rightarrow 16} h(x)$ .
- (c) Compute  $\lim_{x \rightarrow 0} h(x)$ .
- (d) Compute  $\lim_{x \rightarrow -2} h(x)$ .
- (e) State the  $x$ -values at which  $h(x)$  is discontinuous. Justify your statements.

8. Write out the **Limit Definition of the Derivative**  $f'(x)$ .

9. For each of the following functions, find  $f'(x)$  using the limit definition of the derivative.

- (a)  $f(x) = x^4$
- (b)  $f(x) = \sqrt{x}$
- (c)  $f(x) = \frac{1}{x}$
- (d)  $f(x) = \frac{x+1}{x-1}$
- (e)  $f(x) = \frac{1}{\sqrt{x}}$

**Turn in solutions for your group.**