Math 111, Section 01, Fall 2012 ANSWER KEY!!!

Worksheet 1, Thursday, September 6th, 2012

1. Simplify each of the following expressions. Show your work.

   We clear the denominator by flipping and multiplying...

   (a) $$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} \cdot \frac{\left(\frac{d}{c}\right)}{\left(\frac{d}{c}\right)} = \frac{\left(\frac{a}{b}\right)\left(\frac{d}{c}\right)}{1} = \boxed{\frac{ad}{bc}}$$

   (b) $$\frac{1}{\left(\frac{a}{b}\right)} = \frac{1}{\left(\frac{a}{b}\right)} \cdot \frac{\left(\frac{b}{a}\right)}{\left(\frac{b}{a}\right)} = \frac{\left(\frac{b}{a}\right)}{1} = \boxed{\frac{b}{a}}$$

   (c) $$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} \cdot \frac{\left(\frac{1}{c}\right)}{\left(\frac{1}{c}\right)} = \frac{\left(\frac{a}{b}\right) \cdot \left(\frac{1}{c}\right)}{1} = \boxed{\frac{a}{bc}}$$

   (d) $$\frac{a}{\left(\frac{b}{c}\right)} = \frac{a}{\left(\frac{b}{c}\right)} \cdot \frac{\left(\frac{c}{b}\right)}{\left(\frac{c}{b}\right)} = \frac{a \cdot \left(\frac{c}{b}\right)}{1} = \boxed{\frac{ac}{b}}$$

2. Solve each of the following equations (if possible):

   (a) $x^2 - 4x - 21 = 0$

   Factor $(x-7)(x+3) = 0$ means either $x - 7 = 0$ or $x + 3 = 0$. Finally, $\boxed{x = 7}$ or $\boxed{x = -3}$.

   (b) $x^2 - x + 7 = 0$

   Try the Quadratic Formula:

   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Here $a = 1$, $b = -1$ and $c = 7$.

   Then $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(7)}}{2(1)} = \dfrac{1 \pm \sqrt{1 - 28}}{2} = \dfrac{1 \pm \sqrt{-27}}{2}$

   $\boxed{\text{No Real solution}}$ because we have a negative discriminant $(b^2 - 4ac)$.

   (c) $x^2 + 2x - 4 = 0$

   Again, try the Quadratic Formula:

   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Here $a = 1$, $b = 2$ and $c = -4$.

Then $x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2(1)} = \dfrac{-2 \pm \sqrt{4+16}}{2} = \dfrac{-2 \pm \sqrt{20}}{2} = \dfrac{-2 \pm \sqrt{4 \cdot 5}}{2} = \dfrac{-2 \pm \sqrt{4}\sqrt{5}}{2} =$
$\dfrac{-2 \pm 2\sqrt{5}}{2} = \boxed{-1 \pm \sqrt{5}}$

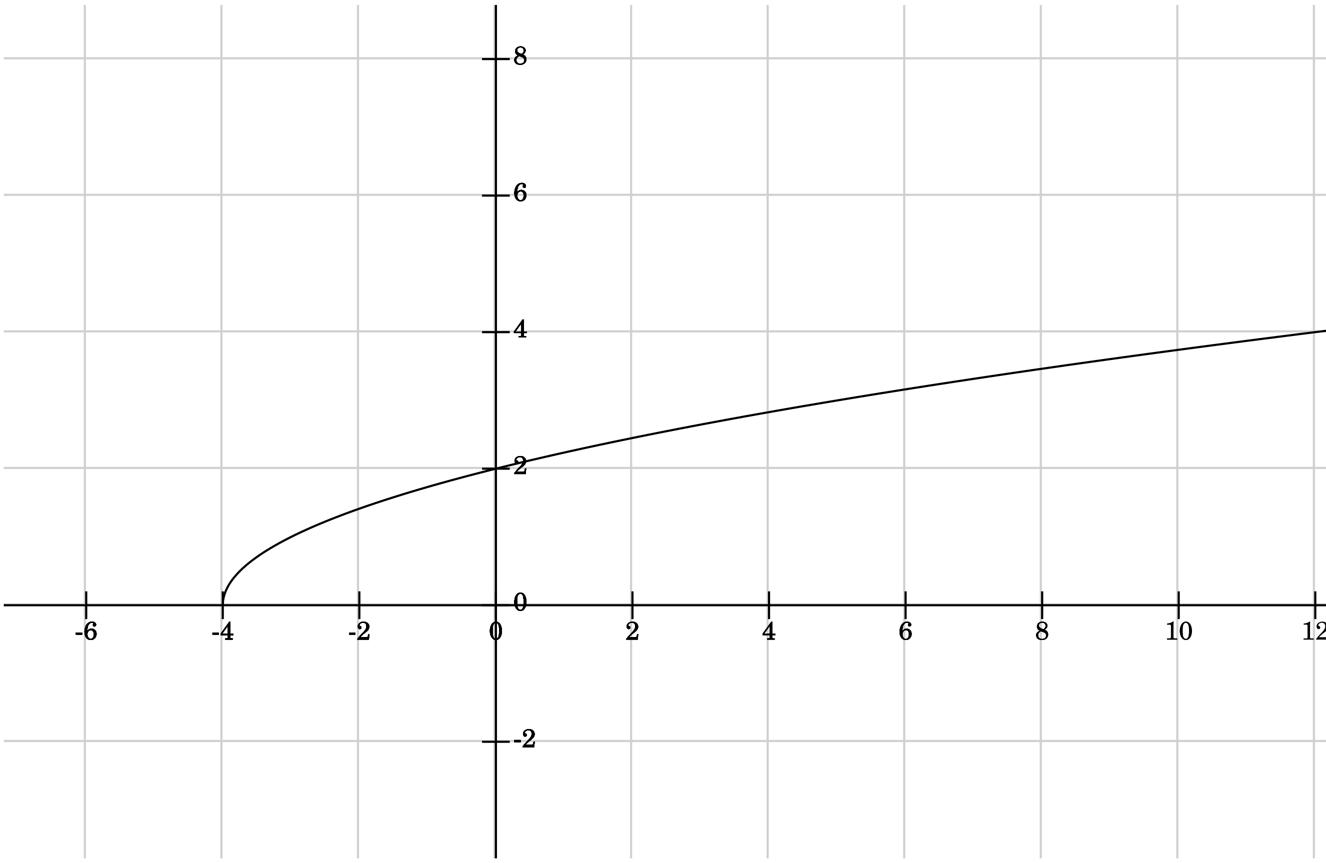
3. YES or NO: Does $\sqrt{x^2 + 4} = x + 2$? Why or why not?

NO, equal functions must take the same value at *every* point. Here test $x = 1$. $\sqrt{5} \neq 3$.

4. Recall from class that we saw the graphs of $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{x}$. Use these graphs to help you do the following:

(a) Sketch the graph of $F(x) = \sqrt{x+4}$. Discuss the Domain and Range for this new function.

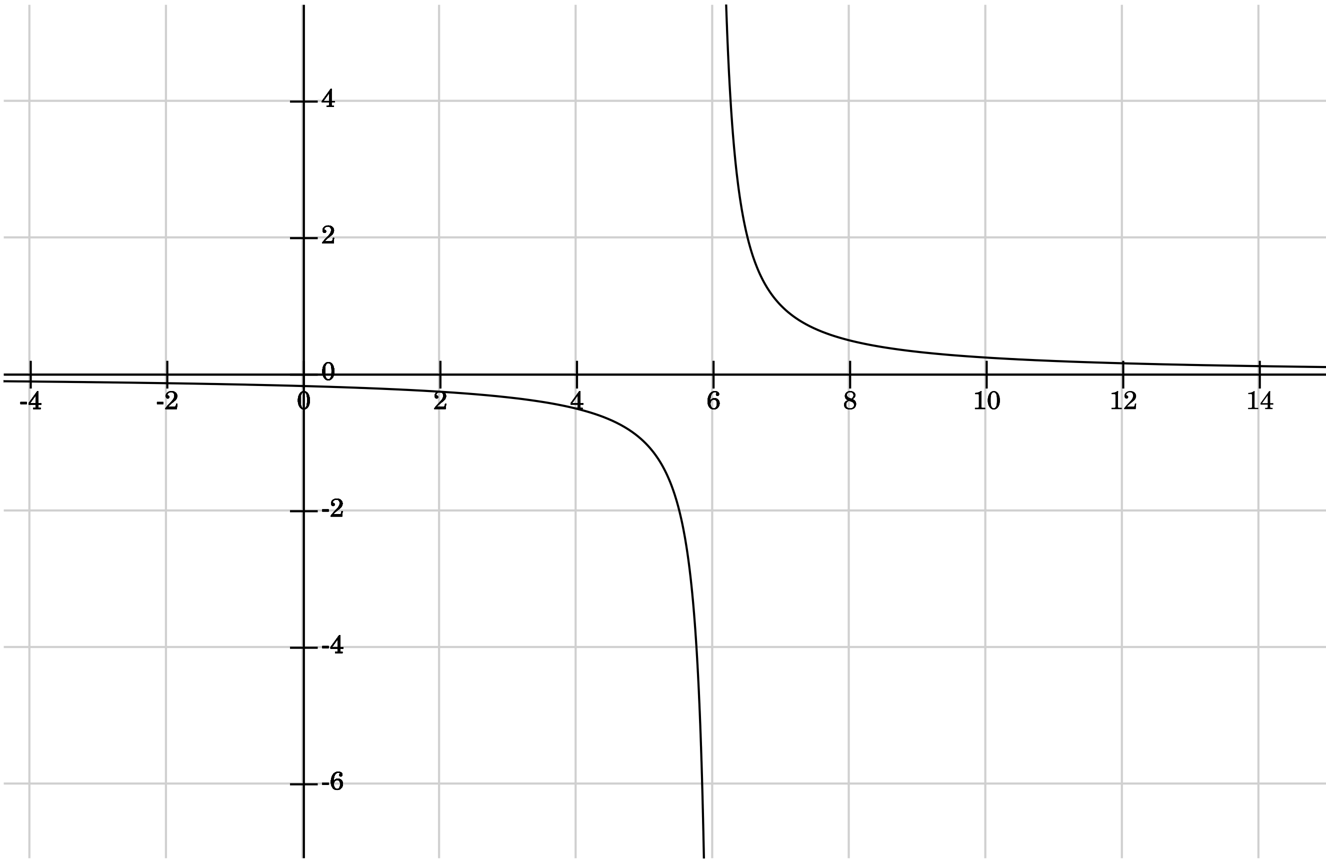
Domain=$\{x : x \geq -4\}$ Range=$[0, \infty) = \{y : y \geq 0\}$



(b) Sketch the graph of $G(x) = \dfrac{1}{x-6}$. Discuss the Domain and Range for this new function. Discuss the output behavior of $G(x)$ as the input value x is near $x = 6$. (Be specific.) Discuss the output behavior of $G(x)$ out near $\pm\infty$.

Domain=$\{x : x \neq 6\}$ Range=$(-\infty, 0) \cup (0, \infty) = \{y : y \neq 0\}$

As x approaches 6 from the positive direction (the right), then function output values are blowing up to ∞. As x approaches 6 from the negative direction (the left), then the function output values are blowing down to $-\infty$. As x approaches $+\infty$ the output values are approaching 0, from the positive direction. (0^+) As x approaches $-\infty$ the output values are approaching 0, but from the negative direction. (0^-)



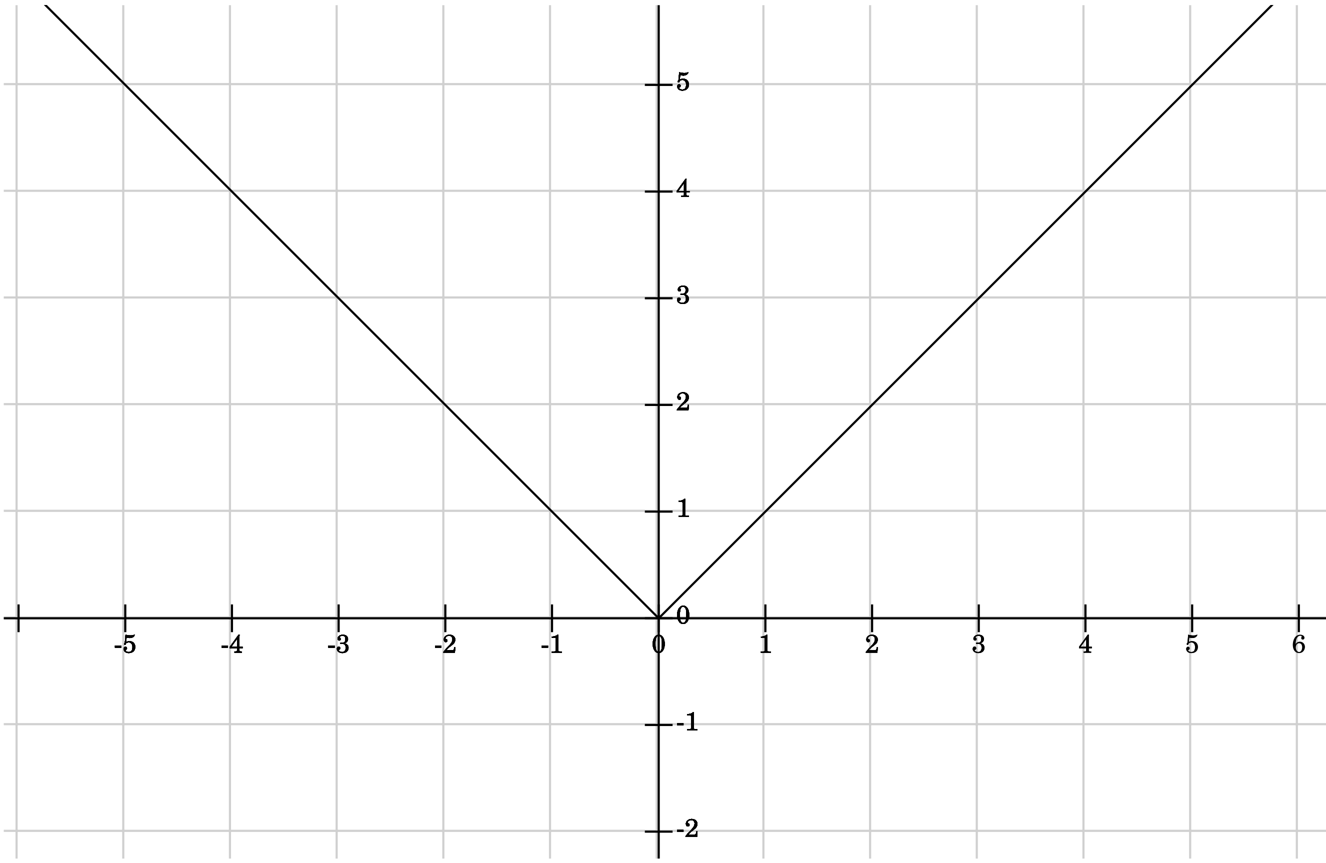
5. The Absolute Value Function $f(x) = |x|$ is a *piece-wise defined function* defined by

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

(a) Give the Domain and Range for this function. Graph the absolute value function. Discuss how this function behaves near $x = 0$.

Domain=$\mathbb{R}$ Range=$\{y : y \geq 0\}$

For $x > 0$ the graph has slope 1 and for $x < 0$ the graph has slope -1. Both pieces of the graph merge together at $x = 0$ with output 0 there.

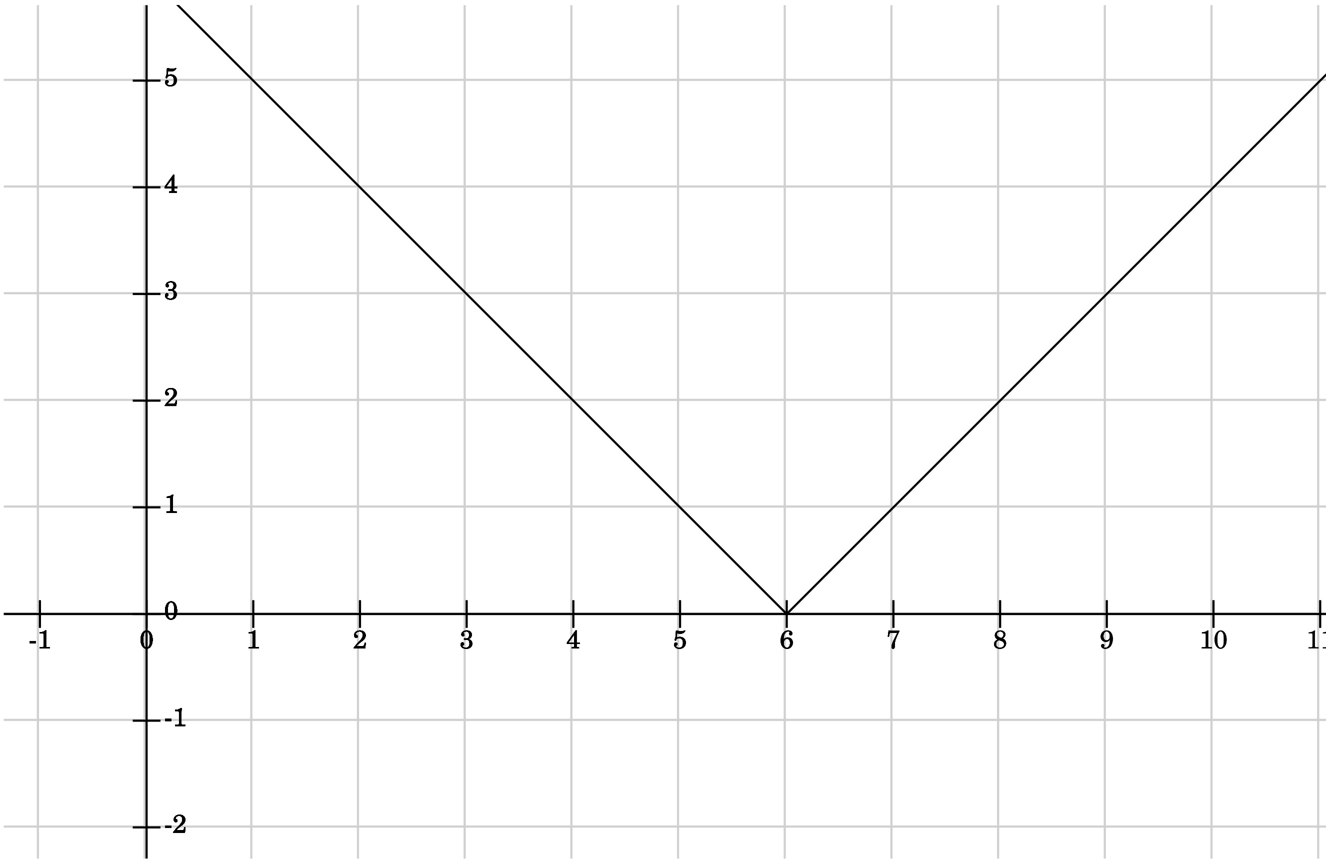


(b) Now consider $g(x) = |x - 6|$. Write out the piece-wise defined definition of this function carefully. THEN use that definition to graph the function g. Discuss how this graph relates to the graph of $f(x) = |x|$. Discuss how this function behaves near $x = 6$.

$$g(x) = |x-6| = \begin{cases} x-6 & \text{if } x-6 \geq 0 \\ -(x-6) & \text{if } x-6 < 0 \end{cases} = \begin{cases} x-6 & \text{if } x \geq 6 \\ 6-x & \text{if } x < 6 \end{cases}$$

Domain=$\mathbb{R}$ Range=$\{y : y \geq 0\}$

This appears to be a shift of the graph $|x|$ to the right 6 units.

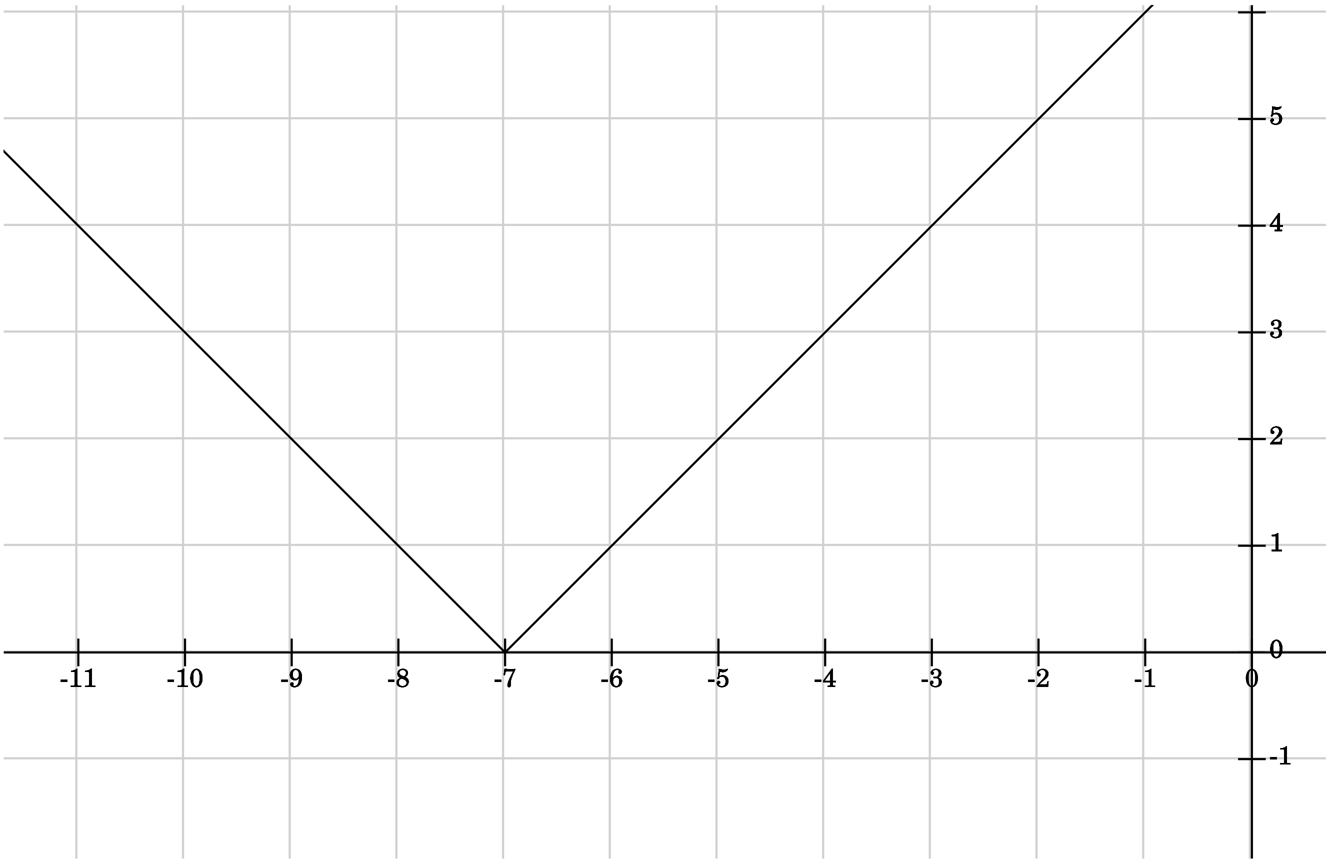


(c) Now consider $h(x) = |x + 7|$. Write out the piece-wise defined definition of this function carefully. THEN use that definition to graph the function h. Discuss how this graph relates to the graph of $f(x) = |x|$. Discuss how this function behaves near $x = -7$.

$$h(x) = |x+7| = \begin{cases} x+7 & \text{if } x+7 \geq 0 \\ -(x+7) & \text{if } x+7 < 0 \end{cases} = \begin{cases} x+7 & \text{if } x \geq -7 \\ -x-7 & \text{if } x < -7 \end{cases}$$

Domain=$\mathbb{R}$	Range=$\{y : y \geq 0\}$

This appears to be a shift of the graph $|x|$ to the left 7 units.



-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
5
4
3
2
1
0
-1

6. Consider the function defined by

$$f(x) = \begin{cases} 2-x & \text{if } x < -1 \\ x+4 & \text{if } -1 < x \leq 1 \\ (x-2)^2 & \text{if } x > 1 \end{cases}$$

Graph $f(x)$ and find its Domain and Range. Discuss the behavior of the function near $x = \pm 1$. Think about how the function behaves as the input values approach $x = 1$ or $x = -1$ *from the left* and *from the right*. (We will formalize this idea soon.)

See me for a sketch.

Domain=$\{x : x \neq -1\}$ Range=$\{y : y \geq 0\}$

It appears that the function is approaching $y = 3$ as the input values approach $x = -1$ from **either** the left or the right. That is from the negative or positive direction. Even though the function is not defined at $x = 3$, the function appears to be approaching $y = 3$.

It appears that the function is approaching $y = 5$ as the input values approach $x = 1$ from the left, and it appears that the function is approaching $y = 1$ as the input values approach $x = 1$ from the left. Nothing formal here yet, we're just starting to study the behavior of the function output values.

7. Let $g(x) = \dfrac{\dfrac{x}{x-1} - \dfrac{x+2}{x}}{x-2}$.

(a) What is the domain of $g(x)$?

Domain $g = \{x : x \neq 0, 1, 2\}$.

(b) Find a simpler formula that agrees with $g(x)$, at least on the domain of g.

$$g(x) = \frac{\dfrac{x}{x-1} - \dfrac{x+2}{x}}{x-2} = \frac{\left(\dfrac{x^2 - (x-1)(x+2)}{x(x-1)}\right)}{x-2}$$
$$= \frac{\left(\dfrac{x^2 - (x^2 + x - 2)}{x(x-1)}\right)}{x-2} = \frac{\left(\dfrac{-x+2}{x(x-1)}\right)}{x-2}$$
$$= \frac{-x+2}{x(x-1)(x-2)} = \frac{-(x-2)}{x(x-1)(x-2)} = \frac{-1}{x(x-1)}.$$

(c) Guess what the behavior of $g(x)$ is near $x = 2$ (even though g is not defined at $x = 2$). How could you do that? (We will prove this in the future.)

$g(x)$ appears to approach $= \dfrac{-1}{2}$. Near $x = 2$ the original function has the same behavior as the simplified version. They are NOT equal functions, since the original one is not defined at $x = 2$, whereas the simplified version is. I didn't ask about the behavior AT $x = 2$, just near $x = 2$.

8. Given two functions f and g. The **Composition** of f and g is defined by

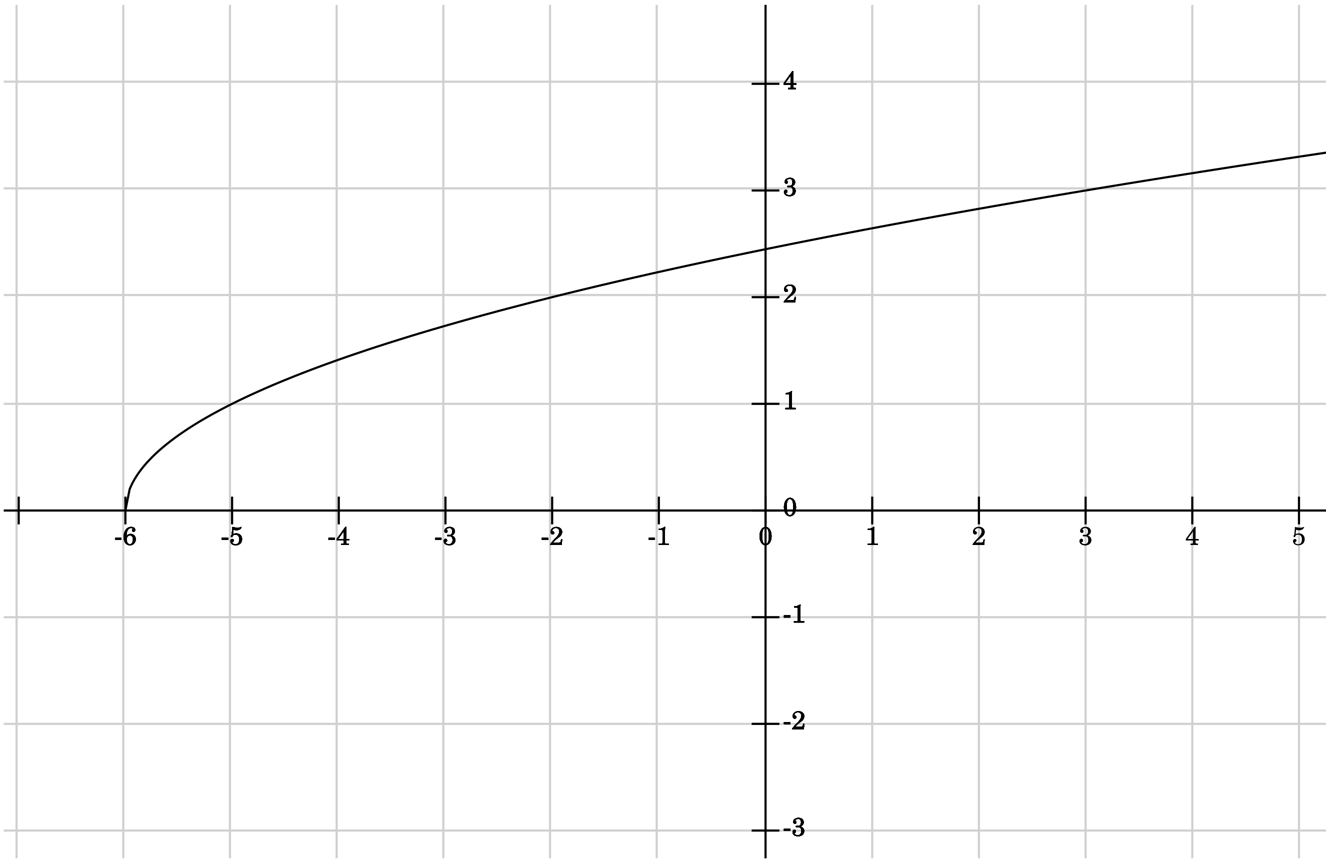
$$f \circ g(x) = f(g(x))$$

(a) Discuss what the Domain of $f \circ g$ is.

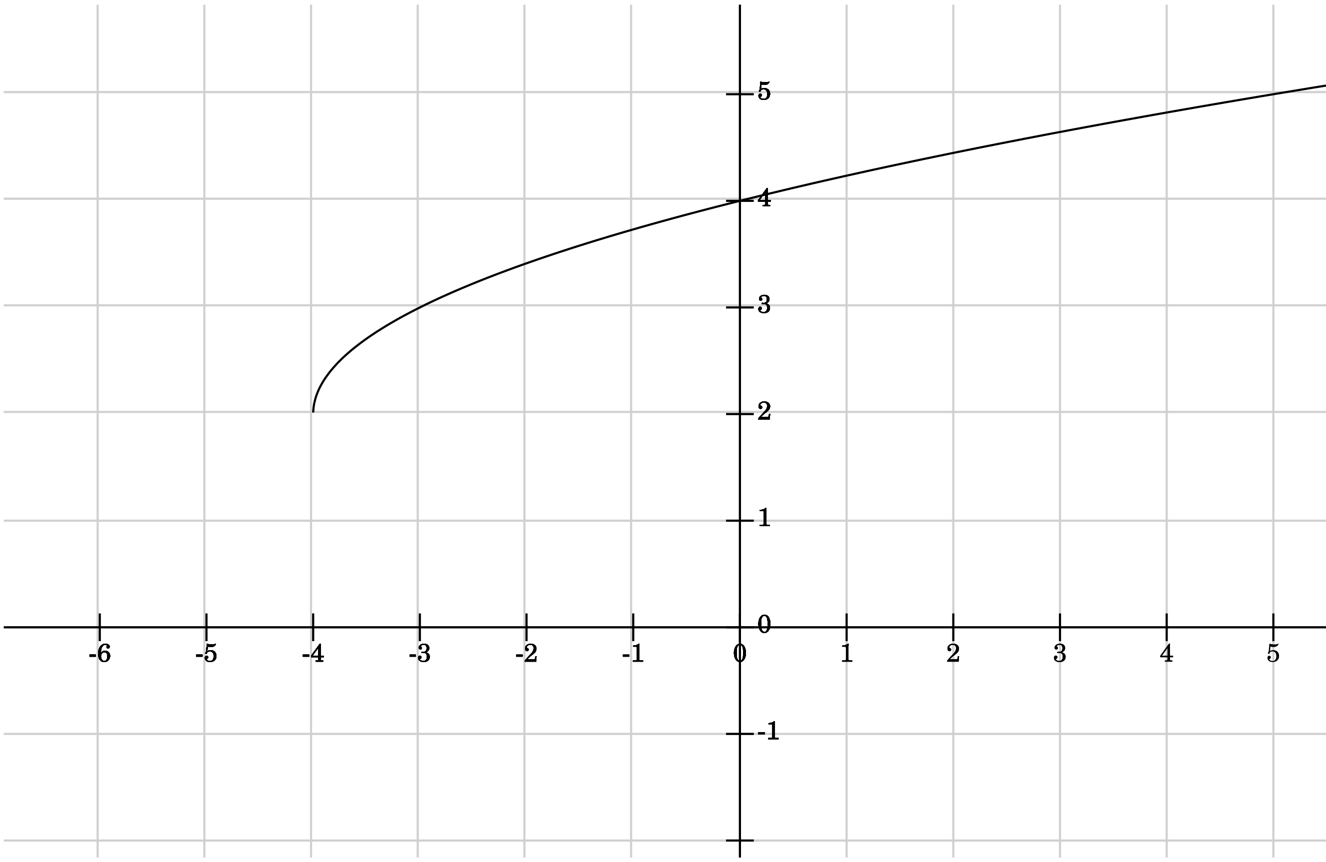
The Domain of the composite function is the set of all x values such that x is in the Domain of g and THEN that output $g(x)$ is in turn in the Domain of f.

(b) Take $f(x) = \sqrt{x+4}$ and $g(x) = x + 2$. Compute and graph both $f \circ g$ and $g \circ f$. Discuss whether or not $f \circ g$ equals $g \circ f$. (Hint: what does itmean for two functions to be equal?)

First, $f \circ g(x) = f(g(x)) = f(x+2) = \sqrt{(x+2)+4} = \boxed{\sqrt{x+6}}$.



Second, $g \circ f(x) = g(f(x)) = g(\sqrt{x+4}) = \boxed{\sqrt{x+4}+2}$.



Notice that these are not the same functions. They don't have the same graphs. Equal functions must take the same value at every element of the domain. Notice that they also don't even have the same domains:

Domain $f \circ g = \{x : x \geq -6\}$ and Domain $g \circ f = \{x : x \geq -4\}$.