Math 111, Section 01, Fall 2012

Worksheet 1, Thursday, September 6th, 2012

1. Simplify each of the following expressions. Show your work.

   (a) $\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}$ (b) $\dfrac{1}{\left(\frac{a}{b}\right)}$

   (c) $\dfrac{\left(\frac{a}{b}\right)}{c}$ (d) $\dfrac{a}{\left(\frac{b}{c}\right)}$

2. Solve each of the following equations (if possible):

   (a) $x^2 - 4x - 21 = 0$ (b) $x^2 - x + 7 = 0$ (c) $x^2 + 2x - 4 = 0$

3. YES or NO: Does $\sqrt{x^2 + 4} = x + 2$? Why or why not?

4. Recall from class that we saw the graphs of $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{x}$. Use these graphs to help you do the following:

   (a) Sketch the graph of $F(x) = \sqrt{x + 4}$. Discuss the Domain and Range for this new function.

   (b) Sketch the graph of $G(x) = \dfrac{1}{x - 6}$. Discuss the Domain and Range for this new function. Discuss the output behavior of $G(x)$ as the input value x is near $x = 6$. (Be specific.) Discuss the output behavior of $G(x)$ out near $\pm\infty$.

5. The Absolute Value Function $f(x) = |x|$ is a *piece-wise defined function* defined by

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

   (a) Give the Domain and Range for this function. Graph the absolute value function. Discuss how this function behaves near $x = 0$.

   (b) Now consider $g(x) = |x - 6|$. Write out the piece-wise defined definition of this function carefully. THEN use that definition to graph the function g. Discuss how this graph relates to the graph of $f(x) = |x|$. Discuss how this function behaves near $x = 6$.

   (c) Now consider $h(x) = |x + 7|$. Write out the piece-wise defined definition of this function carefully. THEN use that definition to graph the function h. Discuss how this graph relates to the graph of $f(x) = |x|$. Discuss how this function behaves near $x = -7$.

6. Consider the function defined piece-wise by

$$f(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ x + 4 & \text{if } -1 < x \leq 1 \\ (x-2)^2 & \text{if } x > 1 \end{cases}$$

Graph $f(x)$ and find its Domain and Range. Discuss the behavior of the function near $x = \pm 1$. Think about how the function behaves as the input values approach $x = 1$ or $x = -1$ *from the left* and *from the right*. (We will formalize this idea soon.)

7. Let $g(x) = \dfrac{\dfrac{x}{x-1} - \dfrac{x+2}{x}}{x-2}$.

   (a) What is the domain of $g(x)$?

   (b) Find a simpler formula that agrees with $g(x)$, at least on the domain of g.

   (c) Guess what the behavior of $g(x)$ is near $x = 2$ (even though g is not defined at $x = 2$). How could you do that? (We will formalize this idea in the future.)

8. Given two functions f and g. The **Composition** of f and g is defined by

$$f \circ g(x) = f(g(x))$$

   (a) Discuss what the Domain of $f \circ g$ is.

   (b) Take $f(x) = \sqrt{x+4}$ and $g(x) = x + 2$. Compute and graph both $f \circ g$ and $g \circ f$. Discuss whether or not $f \circ g$ equals $g \circ f$. (Hint: what does it mean for two functions to be equal?)

Each group turn in solutions for Problems 1-6.

But you do need to understand ALL of these problems.

I will post answer keys on the class webpage.