

Extra Examples of Limits-Answers

1. $\lim_{x \rightarrow 2} \frac{x+7}{x^2-3x+2}$ DNE (Does Not Exist) since RHL \neq LHL
 - RHL: $\lim_{x \rightarrow 2^+} \frac{x+7}{x^2-3x+2} = \lim_{x \rightarrow 2^+} \frac{x+7}{(x-2)(x-1)} = \frac{9}{(0^+) \cdot (1)} = +\infty$
 - LHL: $\lim_{x \rightarrow 2^-} \frac{x+7}{x^2-3x+2} = \lim_{x \rightarrow 2^-} \frac{x+7}{(x-2)(x-1)} = \frac{9}{(0^-) \cdot (1)} = -\infty$

2. $\lim_{x \rightarrow 1} \frac{x+7}{x^2-3x+2}$ DNE since RHL \neq LHL
 - RHL: $\lim_{x \rightarrow 1^+} \frac{x+7}{x^2-3x+2} = \lim_{x \rightarrow 1^+} \frac{x+7}{(x-2)(x-1)} = \frac{8}{(-1) \cdot (0^+)} = -\infty$
 - LHL: $\lim_{x \rightarrow 1^-} \frac{x+7}{x^2-3x+2} = \lim_{x \rightarrow 1^-} \frac{x+7}{(x-2)(x-1)} = \frac{8}{(-1) \cdot (0^-)} = +\infty$

3. $\lim_{x \rightarrow 0} \frac{x+7}{x^2-3x+2}$ $\stackrel{\text{DSP}}{=} \frac{7}{2}$ by Direct Substitution Property (D.S.P.)

4. $\lim_{x \rightarrow -7} \frac{x+7}{x^2+x+1}$ $\stackrel{\text{DSP}}{=} \frac{0}{49-7+1} = \frac{0}{43} = \boxed{0}$ trust this answer!

5. $\lim_{x \rightarrow -7} \frac{x+7}{x^2+2x-35} = \lim_{x \rightarrow -7} \frac{x+7}{(x+7)(x-5)} = \lim_{x \rightarrow -7} \frac{1}{x-5} \stackrel{\text{DSP}}{=} \frac{1}{-7-5} = \frac{1}{-12} = \boxed{-\frac{1}{12}}$

6. $\lim_{x \rightarrow 7} \frac{x+7}{x-7}$ DNE since RHL \neq LHL
 - RHL: $\lim_{x \rightarrow 7^+} \frac{x+7}{x-7} = \frac{14}{0^+} = +\infty$
 - LHL: $\lim_{x \rightarrow 7^-} \frac{x+7}{x-7} = \frac{14}{0^-} = -\infty$

7. $\lim_{x \rightarrow 2} \frac{\sqrt{x+7}-3}{x^2-3x+2} = \lim_{x \rightarrow 2} \frac{\sqrt{x+7}-3}{x^2-3x+2} \cdot \left(\frac{\sqrt{x+7}+3}{\sqrt{x+7}+3} \right) = \lim_{x \rightarrow 2} \frac{(x+7)-9}{(x-2)(x-1)(\sqrt{x+7}+3)}$
 $= \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(x-1)(\sqrt{x+7}+3)} = \lim_{x \rightarrow 2} \frac{1}{(x-1)(\sqrt{x+7}+3)} \stackrel{\text{L.L.}}{=} \frac{1}{1 \cdot (\sqrt{9}+3)} = \boxed{\frac{1}{6}}$

8. $\lim_{x \rightarrow 7} \frac{x+7}{|x-7|} = \boxed{+\infty}$ since RHL=LHL
 - RHL: $\lim_{x \rightarrow 7^+} \frac{x+7}{|x-7|} = \lim_{x \rightarrow 7^+} \frac{x+7}{x-7} = \frac{14}{0^+} = +\infty$
 - LHL: $\lim_{x \rightarrow 7^-} \frac{x+7}{|x-7|} = \lim_{x \rightarrow 7^-} \frac{x+7}{-(x-7)} = \frac{14}{-(0^-)} = +\infty$

9. $\lim_{x \rightarrow -6} \frac{x+7}{|x-7|} = \lim_{x \rightarrow -6} \frac{x+7}{-(x-7)} \stackrel{\text{DSP}}{=} \frac{-6+7}{-(-6-7)} = \boxed{\frac{1}{13}}$

$$10. \lim_{x \rightarrow -7} \frac{\frac{1}{1-x} - \frac{1}{8}}{x+7} = \lim_{x \rightarrow -7} \frac{\frac{8 - (1-x)}{(1-x) \cdot 8}}{x+7} = \lim_{x \rightarrow -7} \frac{7+x}{(1-x) \cdot 8 \cdot (x+7)} = \lim_{x \rightarrow -7} \frac{1}{(1-x) \cdot 8}$$

$$\stackrel{\text{DSP}}{=} \frac{1}{(1 - (-7)) \cdot 8} = \frac{1}{8 \cdot 8} = \boxed{\frac{1}{64}}$$

$$11. \lim_{x \rightarrow 10} \frac{x+7}{|x-7|} = \lim_{x \rightarrow 10} \frac{x+7}{x-7} \stackrel{\text{DSP}}{=} \frac{10+7}{10-7} = \boxed{\frac{17}{3}}$$

$$12. \lim_{x \rightarrow 7} \frac{x^2 - 16x + 63}{x^2 - 14x + 49} = \lim_{x \rightarrow 7} \frac{(x-9)(x-7)}{(x-7)(x-7)} = \lim_{x \rightarrow 7} \frac{x-9}{x-7} \quad \boxed{\text{DNE}} \text{ since RHL} \neq \text{LHL}$$

$$\bullet \text{ RHL: } \lim_{x \rightarrow 7^+} \frac{x-9}{x-7} = \frac{-2}{0^+} = -\infty$$

$$\bullet \text{ LHL: } \lim_{x \rightarrow 7^-} \frac{x-9}{x-7} = \frac{-2}{0^-} = +\infty$$

$$13. \lim_{x \rightarrow 7} \frac{3-x}{7-x} \quad \boxed{\text{DNE}} \text{ since RHL} \neq \text{LHL}$$

$$\bullet \text{ RHL: } \lim_{x \rightarrow 7^+} \frac{3-x}{7-x} = \frac{-4}{0^-} = +\infty$$

$$\bullet \text{ LHL: } \lim_{x \rightarrow 7^-} \frac{3-x}{7-x} = \frac{-4}{0^+} = -\infty$$

$$14. \lim_{x \rightarrow 7} \frac{3-x}{(x-7)^2} = \boxed{-\infty} \text{ since RHL} = \text{LHL}$$

$$\bullet \text{ RHL: } \lim_{x \rightarrow 7^+} \frac{3-x}{(x-7)^2} = \frac{-4}{0^+} = -\infty$$

$$\bullet \text{ LHL: } \lim_{x \rightarrow 7^-} \frac{3-x}{(x-7)^2} = \frac{-4}{0^+} = -\infty$$

$$15. \lim_{x \rightarrow 7} \frac{x^2 - 16x + 63}{x^2 - 49} = \lim_{x \rightarrow 7} \frac{(x-7)(x-9)}{(x-7)(x+7)} = \lim_{x \rightarrow 7} \frac{x-9}{x+7} \stackrel{\text{DSP}}{=} \frac{-2}{14} = \boxed{-\frac{1}{7}}$$

$$16. \lim_{x \rightarrow 7} \frac{7-x}{\sqrt{x+42}-7} = \lim_{x \rightarrow 7} \frac{7-x}{\sqrt{x+42}-7} \cdot \left(\frac{\sqrt{x+42}+7}{\sqrt{x+42}+7} \right) = \lim_{x \rightarrow 7} \frac{(7-x)(\sqrt{x+42}+7)}{(\sqrt{x+42}-7) \cdot (\sqrt{x+42}+7)}$$

$$= \lim_{x \rightarrow 7} \frac{(7-x)(\sqrt{x+42}+7)}{x+42-49} = \lim_{x \rightarrow 7} \frac{(7-x)(\sqrt{x+42}+7)}{x-7} = \lim_{x \rightarrow 7} -(\sqrt{x+42}+7) \stackrel{\text{L.L.}}{=} \boxed{-14}$$

$$17. \lim_{x \rightarrow 7} \frac{x-7}{|x-7|} \quad \text{DNE since RHL} \neq \text{LHL}$$

$$\bullet \text{ RHL: } \lim_{x \rightarrow 7^+} \frac{x-7}{|x-7|} = \lim_{x \rightarrow 7^+} \frac{x-7}{x-7} = 1$$

$$\bullet \text{ LHL: } \lim_{x \rightarrow 7^-} \frac{x-7}{|x-7|} = \lim_{x \rightarrow 7^-} \frac{x-7}{-(x-7)} = -1$$