Review Packet for Exam #3

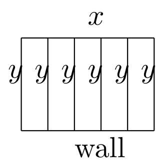
Professor Danielle Benedetto – Math 11

Max-Min Problems

1. Show that of all rectangles with a given area, the one with the smallest perimeter is a square.

2. A rectangle lies in the first quadrant, with one vertex at the orgin, two sides along the coordinate axes, and the fourth vertex on the line $x + 2y - 6 = 0$. Find the maximum area of the rectangle.

3. A farmer wants to use a fence to surround a rectangular field, using an existing stone wall as one side of the plot. She also wants to divide the field into 5 equal pieces using fence parallel to the sides that are perpendicular to the stone wall (see diagram). The farmer must use exactly 1200 feet of fence. What is the maximum area possible for this field?



4. You work for a soup manufacturing company. Your assignment is to design the newest can in the shape of a cylinder. You are given a fixed amount of material, 600 cm^2, to make your can. What are the dimensions of your can which will hold the maximum volume of soup?

5. A rectangular box with square base cost \$ 2 per square foot for the bottom and \$ 1 per square foot for the top and sides. Find the box of largest volume which can be built for \$36.

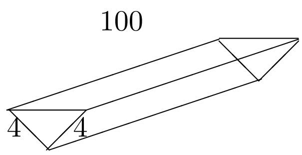
6. Among all the rectangles with given perimeter P, find the one with the maximum area.

7. Consider a cone such that the height is 6 inches high and its base has diameter 6 in. Inside this cone we inscribe a cylinder whose base lies on the base of the cone and whose top intersects the cone in a circle. What is the maximum volume of the cylinder?

8. Consider the right triangle with sides 6, 8 and 10. inside this triangle, we inscribe a rectangle such that one corner of the rectangle is the right angle of the triangle and the opposite corner of the rectangle lies on the hypotenuse. What is the maximum area of the rectangle?

9. A toolshed with a square base and a flat roof is to have volume of 800 cubic feet. If the floor costs \$6 per square foot, the roof \$2 per square foot, and the sides \$5 per square foot, determine the dimensions of the most economical shed.

10. A rectangular sheet of metal 8 inches wide and 100 inches long is folded along the center to form a triangular trough. Two extra pieces of metal are attached to the ends of the trough. The trough is filled with water.

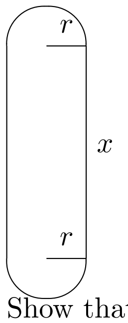


(a). How deep should the trough be to maximize the capacity of the trough?

(b). What is the maximum capacity?

11. A manufacturer wishes to produce rectangular containers with square bottoms and tops, each container having a capacity of 250 cubic inches. The material costs \$2 per square inch for the sides. If the material used for the top and bottom costs twice as much per square inch as the material for the sides, what dimensions will minimize the cost?

12. An outdoor track is to be created in the shape shown and is to have perimeter of 440 yards. Find the dimensions for the track that maximize the area of the rectangular portion of the field enclosed by the track.



13. Show that the entire region enclosed by the outdoor track in the previous example has maximum area if the track is circular.

Initial Valued Differential Equations

14. Find a function $f(x)$ that satisfies $f''(x) = 12x^2 + 5, f'(1) = 5$ and passes through the point $(1, 3)$.

15. Find a function $f(x)$ that satisfies $f''(x) = x + \sin x, f'(0) = 6$ and $f(0) = 4$.

16. Find a function $f(x)$ that satisfies $f''(x) = 2 - 12x, f(0) = 9$ and $f(2) = 15$.

17. Find a function $f(x)$ that satisfies $f''(x) = 20x^3 + 12x^2 + 4, f(0) = 8$ and $f(1) = 5$.

Area and Riemann Sums

18. Evaluate $\displaystyle\int_{-1}^{1} x\ dx$ using Riemann Sums.

19. Evaluate $\displaystyle\int_{0}^{2} x^2 - 5x\ dx$ using Riemann Sums.

20. Use Riemann Sums to estimate $\displaystyle\int_{0}^{1} x^2 + 1\ dx$ using 4 equal-length subintervals and right endpoints.

21. Sand is added to a pile at a rate of $10 + t^2$ cubic feet per hour for $0 \leq t \leq 8$. Compute the Riemann Sum to estimate $\displaystyle\int_{0}^{8} 10 + t^2\ dt$ using 4 subintervals and the left endpoint of each subinterval. Finally, what two things does this Riemann Sum approximate?

22. Compute $\displaystyle\int_{1}^{4} x - 1\ dx$ using three different methods: (a) using Area interpretations of the definite integral, (b) Fundamental Theorem of Calculus, and (c) Riemann Sums.

Differentiation Answer all of the following questions regarding derivatives:

23. Suppose that $e^{xy} + xy = 2$. Compute $\dfrac{dy}{dx}$.

24. $\dfrac{d}{dx} \displaystyle\int_1^x -t - 1 \; dt$

25. $\dfrac{d}{dx} \displaystyle\int_x^7 1 - \sin t \; dt$

26. $\dfrac{d}{dx} \displaystyle\int_0^{\sin x} \sqrt{1 - t^2} \; dt$

27. $\dfrac{d}{dx} \displaystyle\int_{3x}^2 \cos t \; dt$

28. Find $g''(x)$ if $g(x) = \displaystyle\int_{3x}^7 7t^2 + \sin t \; dt$.

29. $\dfrac{d}{dx} \left(\sqrt{\tan x} \cdot \displaystyle\int_0^x t^3 - t \; dt \right)$

30. $\dfrac{d}{dx} \left(\displaystyle\int_0^x t + \sin t \; dt \right)^3$

31. Find $\dfrac{d}{dx} \displaystyle\int_1^{x^2} 3t - 1 \; dt$ using two different methods.

32. Find $\dfrac{d}{dx} \displaystyle\int_x^0 t + 3 \sin t \; dt$ using two different methods.

33. Find $f(x)$ if $\displaystyle\int_1^x f(t) dt = 2x - 2$

34. Find $f(x)$ if $\displaystyle\int_x^0 f(t) dt = x^2$.

35. Differentiate $y = (\sin x) e^{\sqrt{x+2}}$

36. Differentiate $y = (x - e^{-\cos x}) \cdot e^{\frac{x}{2}}$

37. Differentiate $y = e^{e^x} \cdot \cos(e^{\sqrt{x}})$

38. Differentiate $y = \dfrac{1 - e^{-3x}}{x}$

39. Differentiate $y = \dfrac{1 + e^{-2x}}{1 - e^{7x}}$

40. Differentiate $y = \sin(e^x) \cos(e^{-x})$

41. Differentiate $y = (e^{2x} - e^{-3x})^7$

Integration Evaluate each of the following integrals:

42. $\displaystyle\int \frac{1}{\sqrt[3]{(7-5z)^2}}\ dz$

43. $\displaystyle\int_0^{\frac{\pi}{3}} \sec^2\theta\ d\theta$

44. $\displaystyle\int_{-3}^{3} |x^2-1|\ dx$

45. $\displaystyle\int_{-1}^{2} (|x|-4)\ dx$

46. $\displaystyle\int \frac{(x+1)(x+2)}{\sqrt{x}}\ dx$

47. $\displaystyle\int \frac{u+\sqrt{u}+7}{u^3}\ du$

48. $\displaystyle\int_{-3}^{3} x|x|\ dx$

49. $\displaystyle\int_0^{2\pi} |\sin x|\ dx$

50. $\displaystyle\int_0^{\frac{\pi}{6}} \frac{\cos x}{(1+6\sin x)^2}\ dx$

51. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin(2x)\ dx$

52. $\displaystyle\int_0^{\pi} \sin^2\left(\frac{x}{6}\right)\cos\left(\frac{x}{6}\right)\ dx$

53. $\displaystyle\int_0^{\frac{\pi}{8}} \tan^3(2x)\sec^2(2x)\ dx$

54. $\displaystyle\int_{\frac{\pi^2}{36}}^{\frac{\pi^2}{4}} \frac{\cos\sqrt{t}}{\sqrt{t}\sin\sqrt{t}}\ dt$

55. $\displaystyle\int_0^{\frac{\pi}{3}} \frac{3\sin x\cos x}{(1+3\sin^2 x)^2}\ dx$

56. $\displaystyle\int_5^5 3x^5\sqrt{x}\sin(3x)\ dx$

57. $\displaystyle\int (w^3\cos w^4 + 2009)\ dw$

58. $\displaystyle\int \frac{y^3 - 9y\sin y + 26y^{-1}}{y}\ dy$

59. $\int \sqrt{x} \cos(x\sqrt{x})\ dx$

60. $\int \frac{1}{t^2} \sin\left(\frac{1}{t}\right)\ dt$

61. $\int \frac{1}{u^2} \sqrt[3]{1-\frac{1}{u}}\ du$

62. $\int_{-2}^{2} |1-x|\ dx$

63. $\int_{0}^{2\pi} |\cos x - \sin x|\ dx$

64. $\int \frac{2t^5+1}{t^6+3t}\ dt$

65. $\int x(x+1)^{14}\ dx$

66. $\int 7\cos(5x) - 5\sin(7x)\ dx$

67. $\int x\sqrt{2-3x^2}\ dx$

68. $\int x\sqrt{2-3x}\ dx$

69. $\int x(3x-1)^{\frac{5}{7}}\ dx$

70. $\int (x^{\frac{7}{2}} + x^{-\frac{1}{3}})\sqrt{x}\ dx$

71. $\int x^2 e^{-x^3}\ dx$

72. $\int_{1}^{3} x e^{-3x^2}\ dx$

73. $\int \frac{e^{-\frac{1}{x}}}{7x^2}\ dx$

74. $\int \frac{e^x}{(e^x-1)^2}\ dx$

75. $\int \frac{3e^{7x}}{\sqrt{1-e^{7x}}}\ dx$

76. $\int e^{3x} e^{e^{3x}}\ dx$

Area between Curves

77. Compute the area bounded by $y = x^3$ and $y = 4x$.

78. Compute the area bounded by $y = 2|x|$ and $y = 8 - x^2$.

79. Compute the area bounded by $y = 4 - x^2$, $y = x + 2$, $x = -3$, and $x = 0$.

Position, Velocity, Acceleration

80. A ball is thrown upward with a speed of 128 ft/sec from the edge of a cliff 144 ft above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground?

81. The skid marks made by an automobile indicate that its brakes were fully applied for a distance of 90 ft before it came to a stop. Suppose that it is known that the car in question has a constant deceleration of 20 ft/sec^2 under the conditions of the skid. Suppose also that the car was travelling at 60 ft/sec when the brakes were first applied. How long did it take for the car to come to a complete stop?

82. Suppose that a bolt was fired vertically upward from a powerful crossbow at ground level, and that it struck the ground 48 seconds later. If air resistance may be neglected, find the initial velocity of the bolt and the maximum height it reached.

83. Jack throws a baseball straight downward from the top of a tall building. The initial speed of the ball is 25 feet per second. It hits the ground with a speed of 153 feet per second. How tall is the building?

84. A ball is dropped from the top of the building 576 feet high. With what velocity should a second ball be thrown straight downward 3 seconds later so that the two balls hit the ground simultaneously?

85. A particle starts from rest at the point $x = 10$ and moves along the x-axis with acceleration function $a(t) = 12t$. Find its resulting position function.

86. The skid marks made by an automobile indicate that its brakes were fully applied for a distance of 160 ft before it came to a stop. Suppose that it is known that the car in question has a constant deceleration of 20 ft/sec^2 under the conditions of the skid. How fast was the car travelling when its brakes were first applied?

Displacement–Total Distance–Net Change

87. Suppose that the velocity of a moving particle is $v(t) = t^2 - 11t + 24$ feet per second. Find both the displacement and total distance it travels between time $t = 0$ and $t = 10$ seconds.

88. Supppose that water is pumped into an initially empty tank. The rate of water flow into the tank at time t (in seconds) is $50 - t$ liters per second. How much water flows into the tank during the first 30 seconds?

Curve Sketching

89. Use curve sketching techniques to present a detailed sketch for $f(x) = e^{-\frac{x^2}{2}}$.

90. Use curve sketching techniques to present a detailed sketch for $f(x) = (-x^2 + 3x - 3)e^{-x}$.