

Name: _____

Amherst College
DEPARTMENT OF MATHEMATICS
Math 111
Midterm Exam #3
November 30, 2012

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

- You need *not* simplify algebraically complicated answers for the derivative section. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, e^0 should be simplified.

- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		20
2		25
3		10
4		15
5		20
6		10
Total		100

1. [20 Points] **Differentiate** each of the following functions. You **do not** need to simplify your answers.

(a) $f(x) = \int_{\sec x}^7 \sqrt{\cos t + 7e^t} dt$

(b) $f(x) = \tan(e^x + \sqrt{x}) + e^{\tan \sqrt{x}} + \sqrt{e^x + \tan x}$

(c) $f(x) = e^x + x^e + ex + e^e + e^{(e^x)} + (x^e)^e + e^{\frac{1}{x}} - \frac{1}{e^x}$.

2. [25 Points] Compute each of the following integrals. Simplify your answers.

(a) $\int \left(e^{7x} + \frac{1}{e^{4x}} \right)^2 dx$

(b) $\int_0^1 \frac{e^x}{\sqrt{e^x + 8}} dx$

2. [Continued] Compute each of the following integrals. Simplify your answers.

(c) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos x}{\sin^3 x} dx$

(d) $\int x (x - 1)^{\frac{5}{7}} dx$

3. [10 Points] Find the function $f(x)$ that satisfies $f'(x) = \frac{e^{\sqrt{\tan x}} \sec^2 x}{\sqrt{\tan x}}$ and $f\left(\frac{\pi}{4}\right) = 1$.

4. [15 Points] You need to construct a box with a square base with a fixed volume of 24 cubic feet. The material for the bottom and top costs \$3 per square foot, and the material for the sides costs \$1 per square foot. What are the **dimensions** that minimize the cost required to build such a box? What is that **minimum cost**?

(Don't forget to state the common sense bounds, that is, the domain of the function that you are maximizing or minimizing.)

5. [20 Points] Compute $\int_1^3 x^2 - 3x \, dx$ using each of the following **two** different methods:

(a) Fundamental Theorem of Calculus.

(b) Riemann Sums and the limit definition of the definite integral ***.

*** Recall $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{i=1}^n i = \frac{n(n+1)}{2}$, and $\sum_{i=1}^n 1 = n$

6. [10 Points] A moving object has velocity $v(t) = 2t - 6$ feet per second, at time t seconds. Compute the **Total Distance** travelled by this object from time $t = 0$ to $t = 4$ seconds.

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute $\lim_{n \rightarrow \infty} \frac{e^{(1+\frac{1}{n})} + e^{(1+\frac{2}{n})} + e^{(1+\frac{3}{n})} + \dots + e^2}{n}$

OPTIONAL BONUS #2 Compute $\int \sin^3 x \, dx$