Math 11-Benedetto Midterm Exam #2 (Compact Version) March 25, 2011

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers for the derivative section. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$ or $4^{\frac{3}{2}}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

• If you actually read these directions, draw a smiley face at the bottom of the page.

1. [12 Points] Compute each of the following **limits**. Justify your answers.

(a) $\lim_{x\to 0} \dfrac{2\tan(5x)}{7x}$

(b) $\lim_{x\to 0} \dfrac{7x^2 - 8x^3}{\sin^2(3x)}$

(c) $\lim_{x\to \infty} \dfrac{x^2 - x + 5}{3x^7 + x^6 - 2011}$

2. [18 Points] **Differentiate** each of the following functions. You **do not** need to simplify your answers. Please do not waste time simplifying your derivative.

(a) $f(x) = \cos^4\left(\sin\left(\dfrac{7}{x}\right)\right)$

(b) $f(x) = \dfrac{\left(1 - \dfrac{1}{\sqrt{x}}\right)^9}{\sec(4x)}$

(c) $f(x) = \dfrac{1}{\left(\dfrac{1}{x^5} + \sqrt{x^2 - 4}\right)^{\frac{3}{8}}}$

3. [10 Points] Find the **absolute maximum** and **absolute minimum value(s)** of the function

$$F(x) = (x-1)^2(x-9)^2 \qquad \text{on the interval} \qquad [0, 8].$$

4. [25 Points] Let $f(x) = \dfrac{-x^2+x+2}{x^2-2x+1} = \dfrac{-x^2+x+2}{(x-1)^2}$.

For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve. **Hint:**

Take my word for it that (you do **not** have to compute these)

$$\boldsymbol{f'(x) = \dfrac{x-5}{(x-1)^3}} \quad \text{and} \quad \boldsymbol{f''(x) = \dfrac{-2x+14}{(x-1)^4}}.$$

5. [10 Points] Consider the equation $\sin(xy) + \cos y + 7 = \sqrt{xy^3} + 9$. Find $\dfrac{\mathrm{dy}}{\mathrm{dx}}$.

6. [15 Points] The top of a ten foot ladder is sliding down a vertical wall at the rate of one foot every second. Consider the angle formed by the bottom of the ladder and the ground. How fast is this angle changing when the top of the ladder is three feet above the ground?

7. [10 Points] Let $f(x) = \tan^2 x + \cos(2x)$. Find $f'\left(\dfrac{\pi}{6}\right)$.