

Please carefully write all of your answers in your **Blue Book**. Justify all of your answers. There are **No Calculators** allowed.

1. (6 Points) State the **domain** for each of the following functions and justify your answers:

$$(a) \quad h(t) = |t - 1| \qquad (b) \quad f(x) = \frac{5}{x - 7}$$

2. (24 Points) Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

$$(a) \quad \lim_{x \rightarrow 1} \frac{x^2 - 7x}{x^2 - 3x - 11} \qquad (c) \quad \lim_{x \rightarrow 7} \frac{x - 7}{|x - 7|}$$
$$(b) \quad \lim_{x \rightarrow 3} \frac{x^2 - 9}{\sqrt{x + 1} - 2} \qquad (d) \quad \lim_{x \rightarrow 1} \frac{f(x + 1) - 4}{x^2 - x}, \text{ where } f(x) = x^2.$$

3. (15 Points) Prove that $\lim_{x \rightarrow 2} 3 - 4x = -5$ using the $\varepsilon - \delta$ definition of the limit.

4. (15 Points) Suppose that $f(x) = \frac{1}{x - 7}$. Compute $f'(x)$ using the limit definition of the derivative.

5. (10 Points) Suppose that $f(x) = x^2 + x - 6$. Write the *equation* of the tangent line to the curve $y = f(x)$ when $x = 3$. Use the limit definition of the derivative when computing the derivative.

6. (10 Points) Suppose that f and g are functions, and

$$\bullet \lim_{x \rightarrow 5} f(x) = 4 \qquad \bullet \lim_{x \rightarrow 5} g(x) = -7 \qquad \bullet g(x) \text{ is continuous at } x = 5.$$

Evaluate the following quantities and fully justify your answers. Do not just put down a value:

$$(a) \quad \lim_{x \rightarrow 5} (3f(x) - 2g(x)) =$$

$$(b) \quad g(5) =$$

TURN PAPER OVER PLEASE!!

REMEMBER: ALL OF YOUR WORK GOES IN THE BLUE ANSWER BOOK

7. (20 Points) Consider the function defined by

$$f(x) = \begin{cases} -x - 1 & \text{if } x < 0 \\ x^2 - 1 & \text{if } 0 \leq x \leq 3 \\ 10 & \text{if } 3 < x < 7 \\ \frac{1}{x - 7} & \text{if } x > 7 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$. Use this sketch to help answer the following questions:

(b) Compute $\begin{cases} \lim_{x \rightarrow 0^+} f(x) = \\ \lim_{x \rightarrow 0^-} f(x) = \\ \lim_{x \rightarrow 0} f(x) = \end{cases}$

(c) Compute $\begin{cases} \lim_{x \rightarrow 3^+} f(x) = \\ \lim_{x \rightarrow 3^-} f(x) = \\ \lim_{x \rightarrow 3} f(x) = \end{cases}$

(d) Compute $\begin{cases} \lim_{x \rightarrow 7^+} f(x) = \\ \lim_{x \rightarrow 7^-} f(x) = \\ \lim_{x \rightarrow 7} f(x) = \end{cases}$

(e) State the value(s) at which f is discontinuous. Justify your answers using definitions or theorems discussed in class.

BONUS PROBLEM: THIS IS OPTIONAL! Feel free to attempt the following bonus problem, but **ONLY** if you are completely done with the original part of the exam, problems 1-7.

Bonus 1: Let $f(x) = \sqrt{x^3 - 4x^2 + x - 7}$. Compute $f'(x)$ using the limit definition of the derivative.