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Math 111

Review Packet for Exam #1

Limit Practice Problems

Evaluate the following limits. Be clear if the limit Does Not Exist, or is $+\infty$ or $-\infty$. Always justify your work:

1. $\lim_{w \to 0^+} \frac{16}{w} =$

2. $\lim_{w \to 0^-} \frac{16}{w} =$

3. $\lim_{w \to 0} \frac{16}{w} =$

4. $\lim_{t \to 2^+} \frac{3-t}{t-2} =$

5. $\lim_{t \to 2^-} \frac{3-t}{t-2} =$

6. $\lim_{t \to 2} \frac{3-t}{t-2} =$

7. $\lim_{t \to 2^+} \frac{3-t}{(t-2)^2} =$

8. $\lim_{t \to 2^-} \frac{3-t}{(t-2)^2} =$

9. $\lim_{t \to 2} \frac{3-t}{(t-2)^2} =$

10. $\lim_{x \to 4^+} \frac{(x+2)^2}{x^2-3x-4} =$

11. $\lim_{x \to 4^-} \frac{(x+2)^2}{x^2-3x-4} =$

12. $\lim_{x \to 4} \frac{(x+2)^2}{x^2-3x-4} =$

13. $\lim_{x \to 4^+} \frac{x-4}{x^2-3x-4} =$

14. $\lim_{x \to 4^-} \frac{x-4}{x^2-3x-4} =$

15. $\lim_{x \to 4} \frac{x-4}{x^2-3x-4} =$

16. $\lim_{x\to 4^+} \frac{x^2-2x-8}{x^2-3x-4} =$

17. $\lim_{x\to 4^-} \frac{x^2-2x-8}{x^2-3x-4} =$

18. $\lim_{x\to 4} \frac{x^2-2x-8}{x^2-3x-4} =$

19. $\lim_{x\to 6} \frac{x^2-4x-12}{x^2-3x-18} =$

20. $\lim_{x\to 1} \frac{x^2-4x-12}{x^2-3x-18} =$

21. $\lim_{x\to 0} \frac{x^2-4x-12}{x^2-3x-18} =$

22. $\lim_{x\to -3} \frac{x^2-4x-12}{x^2-3x-18} =$

23. $\lim_{x\to -2} \frac{x^2-4x-12}{x^2-3x-18} =$

24. $\lim_{x\to 0+} \frac{x^2-4x-12}{x^2-7x} =$

25. $\lim_{x\to 0-} \frac{x^2-4x-12}{x^2-7x} =$

26. $\lim_{x\to 0} \frac{x^2-4x-12}{x^2-7x} =$

27. $\lim_{x\to 0} \frac{x^2-4x}{x^2-7x} =$

28. $\lim_{x\to 3^+} \frac{x^2-9}{|x-3|} =$

29. $\lim_{x\to 3^-} \frac{x^2-9}{|x-3|} =$

30. $\lim_{x\to 0^+} \frac{x^3+2009x^2+2000x}{|x|} =$

31. $\lim_{x\to 0^-} \frac{x^3+2009x^2+2000x}{|x|} =$

32. $\lim_{x\to (-5)^+} \frac{x^2+6x+5}{|x+5|} =$

33. $\lim_{x\to (-5)^-} \frac{x^2+6x+5}{|x+5|} =$

34. $\lim_{x \to -5} \frac{x^2 + 6x + 5}{|x + 5|} =$

35. $\lim_{t \to 1} \frac{t^2 - 1}{t^2 - 11t + 10} =$

36. $\lim_{t \to 1} \frac{t^2}{t^2 + t - 1} =$

37. $\lim_{t \to -1} \frac{2009(t^2 + 6t + 5)}{t^2 + t} =$

38. $\lim_{x \to 9} \frac{x^2 - 10x + 9}{x^2 + x - 90} =$

39. $\lim_{t \to 1} t^{500} + t^{400} + t^{300} + t^{200} + t^{100} =$

40. $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 2x - 3} =$

41. $\lim_{x \to 1} \frac{x - 1}{\sqrt{x + 3} - 2} =$

42. $\lim_{x \to 9} \frac{9x - x^2}{3 - \sqrt{x}} =$

43. $\lim_{x \to 1} \frac{\sqrt{x^2 + 8} - 3}{x - 1} =$

44. $\lim_{t \to 1} \frac{t + 4}{t^2 + 6t} =$

45. $\lim_{w \to 0} \frac{2}{w + 6} =$

46. $\lim_{w \to 6} \frac{2}{w + 6} =$

47. $\lim_{x \to -5} x^2 - 3x + 6 =$

48. $\lim_{w \to -2} \frac{w + 2}{w^2 - 3w + 2} =$

49. $\lim_{x \to 2} \frac{3}{x - 2} =$

50. $\lim_{x \to -1} \frac{5}{1 - x} =$

51. $\lim_{x \to 1} \frac{5x}{1 - x} =$

52. $\lim_{x \to 5^+} \frac{6x}{5 - x} =$

53. $\lim_{x \to 5^-} \frac{6x}{5 - x} =$

54. $\lim_{x\to 5} \frac{6x}{5-x} =$

55. $\lim_{x\to -4} \frac{x^2-3x-28}{x^2+4x} =$

56. $\lim_{x\to 0} \frac{x^2-3x-28}{x^2+4x} =$

57. $\lim_{x\to 3^-} \frac{-4}{x-3} =$

58. $\lim_{x\to 3} \frac{-4}{x-3} =$

59. $\lim_{x\to 3^+} \frac{-4}{3-x} =$

60. $\lim_{x\to 3} \frac{-4}{3-x} =$

61. $\lim_{x\to 1^+} |x-1|-3 =$

62. $\lim_{x\to 1^-} |x-1|-3 =$

63. $\lim_{x\to 1^+} \frac{x^2-1}{|x-1|} =$

64. $\lim_{x\to 1^-} \frac{x^2-1}{|x-1|} =$

65. $\lim_{x\to 1} \frac{|1-x|}{(1-x)^2} =$

66. $\lim_{x\to 2} \frac{x^2-4}{|x-2|} =$

67. $\lim_{x\to 7^-} \frac{7-x}{|x-7|} =$

68. $\lim_{x\to 0^-} \frac{x}{x-|x|} =$

69. $\lim_{x\to 2^+} \frac{2-x}{|x-2|} =$

70. $\lim_{x\to 3} \frac{\sqrt{x+6}-3}{x^2-x-6} =$

71. $\lim_{x\to 7} \frac{\frac{1}{7}-\frac{1}{x}}{x-7} =$

72. $\lim_{x\to -6} \frac{\frac{1}{2-x}-\frac{1}{8}}{x+6} =$

73. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{3-x} =$

74. $\lim_{x\to 7} \frac{x^2-49}{2-\sqrt{x-3}} =$

75. $\lim_{x\to 5} \frac{\frac{1}{\sqrt{x+20}}-\frac{1}{5}}{x-5} =$

Use the $\varepsilon - \delta$ definition for limits to prove each of the following:

76. $\lim_{x\to 2} 7x-6 = 8.$

77. $\lim_{x\to -7} 2-\frac{3}{7}x = 5.$

78. $\lim_{x\to -2} 2x+1 = -3.$

79. $\lim_{x\to 3} 1-4x = -11.$

80. $\lim_{t\to 2} 5-4t = -3.$

81. $\lim_{x\to -1} 4-3x = 7.$

82. $\lim_{t\to 2} -2t-5 = -9.$

83. $\lim_{x\to 4} -3x+17 = 5.$

84. $\lim_{x\to -3} 1-5x = 16.$

85. $\lim_{x\to -14} \frac{4}{7}x+3 = -5.$

Tangent Lines Please use the limit definition for the derivative when computing the derivatives in this section.

86. Find an equation for the tangent line to the graph of $f(x) = x - 2x^2$ at the point $(1, -1)$

87. Find an equation for the tangent line to the graph of $f(x) = \sqrt{x}$ at $x = 4$

88. At which point(s) does the graph of $f(x) = -x^2 + 13$ have a horizontal tangent line?

89. At which point(s) of the graph of $f(x) = -x^3 + 13$ is the slope of the tangent line equal to -27? What's the picture representing this problem?

90. There are two points on the graph of the curve $y = -x^2 + 7$ whose tangent line to the graph at those points passes through the point $(0, 11)$. Find those two points.

91. Find the equation of the line passing through $(2, 3)$ which is perpendicular to the tangent to the curve $y = x^3 - 3x + 1$ at the point $(2, 3)$.

92. Find the equation of the tangent line to the curve $y = x^3 + x$ at the point(s) where the slope equals 4.

Derivatives Use the limit definition of the derivative to calculate the derivative for each of the following functions:

93. $f(x) = 3 - 9x^2$

94. $f(x) = -4x - x^2 - 3$

95. $f(x) = \dfrac{-3}{x}$

96. $f(x) = -9x^2 + 3$

97. $f(x) = x^3$

98. $f(x) = x^2 - 4x + 3$

99. $f(x) = \dfrac{1}{x^2}$

100. $f(x) = \sqrt{x - 7}$

101. $f(x) = \sqrt{3x - 7}$

102. $f(x) = \dfrac{1}{x^3}$

103. $f(x) = \dfrac{1}{\sqrt{x}}$

Functions and Limit Practice Problems Evaluate the following limits:

104. Let $g(x) = 2x + 1$. Compute $\displaystyle\lim_{x \to 1} \frac{x - 1}{g(x^2) - 3} =$

105. Let $G(u) = u^2 + u$. Compute $\displaystyle\lim_{u \to 2} \frac{u^2 - 2u}{G(u - 3)} =$

106. Let $F(x) = |x| + 1$. Compute $\displaystyle\lim_{x \to 4} \frac{F(x - 1)}{F(x - 5)} =$

107. Let $h(y) = y^2 - 3$. Compute $\displaystyle\lim_{x \to -2} \frac{x + 2}{h(2x) - h(x + 6)} =$

108. Let $g(x) = \sqrt{x}$. Compute $\displaystyle\lim_{s \to 1} \frac{g(s^2 + 8) - 3}{s - 1} =$

109. Let $f(t) = \dfrac{1}{t}$. Compute $\displaystyle\lim_{t \to 2} \frac{f(t - 1) - 2f(t)}{t^2 - 4} =$

More Tangent Lines Please use the limit definition for the derivative when computing derivatives in this section.

110. Find an equation for the tangent line to the graph of $f(x) = \dfrac{1}{x-1}$ at the point $(0, -1)$.

111. Find an equation for the tangent line to the graph of $f(x) = \dfrac{1}{x+1}$ at the point $\left(1, \dfrac{1}{2}\right)$.

112. Find an equation for the tangent line to the graph of $y = \dfrac{3}{x} + 1$ when $x = 1$.

113. Find an equation for the tangent line to the graph of $y = x^2 - 4x + 2$ when $x = 1$. At what point is the tangent line to this curve horizontal?

Functions Please state what the domain is for each of the following functions.

114. $f(x) = \dfrac{x+2}{x-1}$

115. $g(x) = \sqrt{x-2}$

116. $g(x) = \sqrt{2-x}$

117. $g(x) = \dfrac{1}{\sqrt{2-x}}$

118. $f(x) = \dfrac{x-3}{x^2+3}$

119. $w(x) = \dfrac{1}{x-4}$

120. $f(x) = \dfrac{x^2+6x+8}{x+2}$

More Functions

121. Let $g(x) = \dfrac{x+1}{x}$. Compute (and simplify, if possible) the following:

    (a) $g(2) =$
    (b) $g(0) =$
    (c) $g(\frac{1}{2}) =$
    (d) $g(\frac{1}{10}) =$
    (e) $g(t-2) =$
    (f) $\dfrac{g(2+h) - g(2)}{h} =$

122. Let $f(x) = \dfrac{1}{x+1} - \dfrac{1}{x}$. Compute (and simplify, if possible) the following:

    (a) $f(1) =$
    (b) $f(-1) =$
    (c) $f(-\frac{1}{2}) =$
    (d) $f(t-1) =$
    (e) $f(\frac{1}{t}) =$

More Functions

123. Let $f(x) = \sqrt{x}$, $g(x) = x^2 + 4$, and $h(x) = \dfrac{1}{x}$. Compute (and simplify, if possible) the following:

(a) $f \circ g(x) =$

(b) $g \circ f(x) =$

(c) $g \circ h(x) - h \circ g(x) =$

(d) $f \circ h(x) - h \circ f(x) =$

(e) $h \circ g \circ f(x) =$

(f) $g \circ f \circ f(x) =$

(g) $g \circ g(x) =$

124. Suppose $\lim_{x\to 3} f(x) = 3$ and $\lim_{x\to 3} g(x) = -2$. Assume g is continuous at $x = 3$. Find each of the following values. Justify your work.

(a) $\lim_{x\to 3} 2f(x) - 4g(x) =$

(b) $\lim_{x\to 3} g(x) \cdot \dfrac{x^2 - 9}{x - 3} =$

(c) $g(3) =$

(d) $\lim_{x\to 3} g(f(x)) =$

(e) $\lim_{x\to 3} \sqrt{(f(x))^2 - 8g(x)} =$

Consider each of the following piecewise defined functions. Answer the related questions. *Justify* your answers please.

125. Let $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 2 \\ 8 - x & \text{if } x > 2 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x\to 2} f(x) =$

$\lim_{x\to 0} f(x) =$

126. Let $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \le x \le 1 \\ 3 - x & \text{if } x > 1 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x\to 2} f(x) =$

$\lim_{x\to 1} f(x) =$

$\lim_{x\to 0} f(x) =$

127. Let $f(x) = \begin{cases} \dfrac{1}{x-4} & \text{if } x < 2 \\ \dfrac{1}{x} & \text{if } x \geq 2 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x \to 1} f(x) =$

$\lim_{x \to 2} f(x) =$

128. Let $f(x) = \begin{cases} -3x + 4 & \text{if } x \leq 3 \\ -2 & \text{if } x > 3 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x \to 3} f(x) =$

$\lim_{x \to -2} f(x) =$

129. Let $f(t) = \begin{cases} t - 3 & \text{if } t \leq 3 \\ 3 - t & \text{if } 3 < t < 5 \\ 1 & \text{if } t = 5 \\ 3 - t & \text{if } t > 5 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{t \to 3} f(t) =$

$\lim_{t \to 0} f(t) =$

$\lim_{t \to 5} f(t) =$

130. Let $f(x) = \begin{cases} -2x & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x \leq 2 \\ 6 - x & \text{if } x > 2 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x \to -2} f(x) =$

$\lim_{x \to 0} f(x) =$

$\lim_{x \to 2} f(x) =$

$\lim_{x \to 6} f(x) =$

131. Let $f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ x & \text{if } -1 < x < 1 \\ 1 - x & \text{if } x \geq 1 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x \to -1} f(x) =$

$\lim_{x \to 1} f(x) =$

132. Let $f(x) = \begin{cases} x-1 & \text{if } x < 2 \\ 1 & \text{if } 2 < x < 4 \\ 3 & \text{if } x = 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x \to 0} f(x) =$

$\lim_{x \to 2} f(x) =$

$\lim_{x \to 4} f(x) =$

$f(4) =$

133. Let $h(x) = \begin{cases} \dfrac{8}{x+2} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \dfrac{1}{2}x - 4 & \text{if } 0 < x \leq 16 \\ \sqrt{x} & \text{if } x > 16 \end{cases}$

Sketch the graph. Find the numbers at which h is discontinuous. Evaluate:

$\lim_{x \to -2} h(x) =$

$\lim_{x \to 0} h(x) =$

$\lim_{x \to 16} h(x) =$

134. Let $h(x) = \begin{cases} \dfrac{8}{x-2} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \dfrac{1}{2}x - 4 & \text{if } x > 0 \end{cases}$

Sketch the graph. Find the numbers at which h is discontinuous. Evaluate:

$\lim_{x \to 0} h(x) =$

$\lim_{x \to 2} h(x) =$

135. Let $h(x) = \begin{cases} \dfrac{8}{x-2} & \text{if } x \leq 0 \\ \\ \dfrac{1}{2}x - 4 & \text{if } x > 0 \end{cases}$

Sketch the graph. Find the numbers at which h is discontinuous. Evaluate:

$\lim_{x \to 0} h(x) =$

$\lim_{x \to 1} h(x) =$

$h(0) =$