• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [30 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle \lim_{x \to -7} \frac{x^2 + 5x - 14}{x^2 - 4x + 4} =$

(b) $\displaystyle \lim_{x \to 4} \frac{x^2 - 9x + 20}{|4 - x|} =$

(c) $\displaystyle \lim_{x \to -6} \frac{f(x^2) + 5x - 8}{[f(x)]^2 + 5x + 14} =$ where $f(x) = x + 2$

(d) $\displaystyle \lim_{x \to 2} \frac{x^2 + 5x - 14}{x^2 - 4x + 4} =$

(e) $\displaystyle \lim_{x \to 8} \frac{3 - \sqrt{x + 1}}{x^2 - 7x - 8} =$

2. [13 Points]   Prove that $\displaystyle \lim_{x \to 5} 7 - 2x = -3$ using the $\varepsilon - \delta$ definition of the limit.

3. [15 Points]   Suppose that $f(x) = \dfrac{x + 7}{x - 3}$.   Compute $f'(x)$ using the **limit definition of the derivative**.

4. [10 Points] Suppose that $f(x) = 5 - 7x + 4x^2 - x^3$.   Write the **equation of the tangent line** to the curve $y = f(x)$ when $x = 1$. **Use the limit definition of the derivative when computing the derivative.**

5. [6 Points] Suppose that f and g are functions, **and**

- $\displaystyle \lim_{x \to 3} f(x) = 9$ • $\displaystyle \lim_{x \to 7} g(x) = -6$ • $\displaystyle \lim_{x \to 4} f(x) = 7$
- $g(x)$ is continuous at $x = 7$.   • $f(x)$ is continuous at $x = 4$.

(a) Compute $g \circ f(4) =$ (Do **not** just put down a value. Justify your answer.)

(b) Does $f(3) = 9$?   Why or why not?

6. [6 Points] Suppose that $f(x) = \sqrt{x + 4}$ and $g(x) = x + 2$.
(a) Compute **and** graph $f \circ g(x)$.   **(b)** Compute **and** graph $g \circ f(x)$.

7. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-7} & \text{if } x > 7 \\ 1 & \text{if } x = 7 \\ 7-x & \text{if } 0 < x < 7 \\ 16 - x^2 & \text{if } -4 < x \le 0 \\ \dfrac{1}{x+4} & \text{if } x < -4 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the **Domain** of the function $f(x)$.

(c) Compute $\begin{cases} \lim\limits_{x \to 0^+} f(x) = \\ \lim\limits_{x \to 0^-} f(x) = \\ \lim\limits_{x \to 0} f(x) = \end{cases}$ **(d)** Compute $\begin{cases} \lim\limits_{x \to 7^+} f(x) = \\ \lim\limits_{x \to 7^-} f(x) = \\ \lim\limits_{x \to 7} f(x) = \end{cases}$ **(e)** Compute $\begin{cases} \lim\limits_{x \to -4^+} f(x) = \\ \lim\limits_{x \to -4^-} f(x) = \\ \lim\limits_{x \to -4} f(x) = \end{cases}$

(f) State the value(s) at which f is discontinuous. Justify your answer(s) using definitions or theorems discussed in class.

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute $\lim\limits_{x \to 2} \dfrac{(4 - \sqrt{x+14})(\sqrt{13 - x^2} - 3)}{(6 - \sqrt{40 - 2x})(\sqrt{x^2 + 21} - 5)} =$

OPTIONAL BONUS #2 Let $f(x) = \sqrt{\dfrac{x^2 + 1}{7 - x^3}}$. Compute $f'(x)$.

OPTIONAL BONUS #3 Compute $\lim\limits_{x \to 0} \dfrac{|x-1| - |x+1| - |x|}{|x| + |2-x| - |x+2|} =$

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