

Name: _____

Amherst College
DEPARTMENT OF MATHEMATICS
Math 111
Section 01
Midterm Exam #1
September 28, 2012

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		30
2		13
3		15
4		10
5		6
6		6
7		20
Total		100

1. [30 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x \rightarrow -7} \frac{x^2 + 5x - 14}{x^2 - 4x + 4} =$

(b) $\lim_{x \rightarrow 4} \frac{x^2 - 9x + 20}{|4 - x|} =$

(c) $\lim_{x \rightarrow -6} \frac{f(x^2) + 5x - 8}{[f(x)]^2 + 5x + 14} =$ where $f(x) = x + 2$

1. (Continued) Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(d) $\lim_{x \rightarrow 2} \frac{x^2 + 5x - 14}{x^2 - 4x + 4} =$

(e) $\lim_{x \rightarrow 8} \frac{3 - \sqrt{x+1}}{x^2 - 7x - 8} =$

2. [13 Points] Prove that $\lim_{x \rightarrow 5} 7 - 2x = -3$ using the $\varepsilon - \delta$ definition of the limit.

3. [15 Points] Suppose that $f(x) = \frac{x+7}{x-3}$. Compute $f'(x)$ using the **limit definition of the derivative**.

4. [10 Points] Suppose that $f(x) = 5 - 7x + 4x^2 - x^3$. Write the **equation of the tangent line** to the curve $y = f(x)$ when $x = 1$.

Use the limit definition of the derivative when computing the derivative.

5. [6 Points] Suppose that f and g are functions, **and**

- $\lim_{x \rightarrow 3} f(x) = 9$
- $\lim_{x \rightarrow 7} g(x) = -6$
- $\lim_{x \rightarrow 4} f(x) = 7$
- $g(x)$ is continuous at $x = 7$.
- $f(x)$ is continuous at $x = 4$.

(a) Compute $g \circ f(4) =$ (Do **not** just put down a value. Justify your answer.)

(b) Does $f(3) = 9$? Why or why not?

6. [6 Points] Suppose that $f(x) = \sqrt{x+4}$ and $g(x) = x+2$.

(a) Compute **and** graph $f \circ g(x)$.

(b) Compute **and** graph $g \circ f(x)$.

7. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-7} & \text{if } x > 7 \\ 1 & \text{if } x = 7 \\ 7-x & \text{if } 0 < x < 7 \\ 16-x^2 & \text{if } -4 < x \leq 0 \\ \frac{1}{x+4} & \text{if } x < -4 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the **Domain** of the function $f(x)$.

7. (Continued) Continue to consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-7} & \text{if } x > 7 \\ 1 & \text{if } x = 7 \\ 7-x & \text{if } 0 < x < 7 \\ 16-x^2 & \text{if } -4 < x \leq 0 \\ \frac{1}{x+4} & \text{if } x < -4 \end{cases}$$

(c) Compute $\begin{cases} \lim_{x \rightarrow 0^+} f(x) = \\ \lim_{x \rightarrow 0^-} f(x) = \\ \lim_{x \rightarrow 0} f(x) = \end{cases}$

(d) Compute $\begin{cases} \lim_{x \rightarrow 7^+} f(x) = \\ \lim_{x \rightarrow 7^-} f(x) = \\ \lim_{x \rightarrow 7} f(x) = \end{cases}$

(e) Compute $\begin{cases} \lim_{x \rightarrow -4^+} f(x) = \\ \lim_{x \rightarrow -4^-} f(x) = \\ \lim_{x \rightarrow -4} f(x) = \end{cases}$

(f) State the value(s) at which f is discontinuous. Justify your answer(s) using definitions or theorems discussed in class.

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute $\lim_{x \rightarrow 2} \frac{(4 - \sqrt{x + 14})(\sqrt{13 - x^2} - 3)}{(6 - \sqrt{40 - 2x})(\sqrt{x^2 + 21} - 5)} =$

OPTIONAL BONUS #2 Let $f(x) = \sqrt{\frac{x^2 + 1}{7 - x^3}}$. Compute $f'(x)$.

OPTIONAL BONUS #3 Compute $\lim_{x \rightarrow 0} \frac{|x - 1| - |x + 1| - |x|}{|x| + |2 - x| - |x + 2|} =$