The Substitution Technique

The substitution technique applies to both definite and indefinite integrals, but it works slightly differently in each case. It is ***very*** important to respect these differences.

Indefinite Integrals

Key points to note:

- There are *no* limits of integration to worry about.
- The original variable ***ALWAYS*** reappears.

Here is an example:

$$\underbrace{\int \sin^2 x \cos x \, dx}_{u = \sin x,\ du = \cos x\, dx} \;=\; \int u^2 \, du \;=\; \tfrac{1}{3}u^3 + C \;=\; \underbrace{\tfrac{1}{3}\sin^3 x + C}_{x \text{ reappears}}$$

Definite Integrals

Key points to note:

- The *variables* and *limits of integration* change *simultaneously.*
- Once you switch to u, the original variable ***NEVER*** reappears.

Here is an example:

$$\underbrace{\int_0^{\pi/2} \sin^2 x \cos x \, dx}_{u = \sin x,\ du = \cos x\, dx} = \int_{\underset{x=0 \Rightarrow u=\sin(0)=0}{0}}^{\overset{x=\pi/2 \Rightarrow u=\sin(\pi/2)=1}{1}} u^2 \, du \;=\; \underbrace{\tfrac{1}{3}u^3\Big|_0^1 = \tfrac{1}{3}(1^3 - 0^3) = \tfrac{1}{3}}_{x \text{ never reappears!}}$$

Common Error

I often see solutions that look like this:

$$\int_0^{\pi/2} \sin^2 x \cos x \, dx = \int_0^{\pi/2} u^2 \, du = \tfrac{1}{3}u^3\Big|_0^{\pi/2} = \tfrac{1}{3}\sin^3 x\Big|_0^{\pi/2} = \tfrac{1}{3}(\sin^3(\tfrac{\pi}{2}) - \sin^3(0)) = \tfrac{1}{3}$$

Even though the number $\frac{1}{3}$ is the correct number, its correctness does not follow from this computation since the computation includes two equal signs that are false. Hence this solution is ***wrong***. Be sure you know why!!! (Hint: $\int_0^{\pi/2} u^2 \, du = \frac{\pi^3}{24}$.)