Math 111 Fall 2014 Worksheet #11 Answer Key

1a. $$\lim_{x\to5} \frac{5-x}{\sqrt{x+4}-3}\cdot\left(\frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}\right) = \lim_{x\to5}\frac{(5-x)(\sqrt{x+4}+3)}{(x+4)-9}$$

$$= \lim_{x\to5}\frac{-\cancel{(x-5)}(\sqrt{x+4}+3)}{\cancel{x-5}} = \lim_{x\to5} -(\sqrt{x+4}+3) = -(3+3) = \boxed{-6}$$

b. $$\lim_{x\to2}\frac{g(x^2)+x-3}{[g(x+1)]^2-x+2} \quad \text{where } g(x)=x-3$$

$$= \lim_{x\to2}\frac{(x^2-3)+x-3}{(\underbrace{x+1-3}_{x-2})^2-x+2} = \lim_{x\to2}\frac{x^2+x-6}{x^2-4x+4-x+2}$$

$$= \lim_{x\to2}\frac{x^2+x-6}{x^2-5x+6} \overset{0/0}{=} \lim_{x\to2}\frac{(x+3)\cancel{(x-2)}}{(x-3)\cancel{(x-2)}} \overset{DSP}{=} \frac{5}{-1} = \boxed{-5}$$

c. $$\lim_{x\to1}\frac{x^2-8x+7}{x^2-2x+1} = \lim_{x\to1}\frac{(x-7)\cancel{(x-1)}}{(x-1)\cancel{(x-1)}} \quad {-6}/{0}$$

RHL: $$\lim_{x\to1^+}\frac{x-7}{x-1} = \frac{-6}{0^+} = -\infty$$

LHL: $$\lim_{x\to1^-}\frac{x-7}{x-1} = \frac{-6}{0^-} = +\infty$$

$\boxed{\text{DNE b/c RHL} \neq \text{LHL}}$

1d. $\lim_{x \to 5} \frac{x^2-4x-5}{|5-x|}$ $\frac{0}{0}$

$|5-x| = \begin{cases} 5-x \text{ if } 5-x \geq 0 & x<5 \quad \text{LHL} \\ -(5-x) \text{ if } 5-x<0 & x>5 \quad \text{RHL} \end{cases}$

$\boxed{\text{DNE b/c RHL} \neq \text{LHL}}$

RHL: $\lim_{x \to 5^+} \frac{x^2-4x-5}{|5-x|} = \lim_{x \to 5^+} \frac{(x-5)(x+1)}{-(5-x)} = \lim_{x \to 5^+} \frac{\cancel{(x-5)}(x+1)}{\cancel{x-5}}$

$= 6$

LHL: $\lim_{x \to 5^-} \frac{x^2-4x-5}{|5-x|} = \lim_{x \to 5^-} \frac{(x-5)(x+1)}{5-x} = \lim_{x \to 5^-} \frac{\cancel{(x-5)}(x+1)}{-\cancel{(x-5)}}$

$= -6$

2. $\lim_{x \to 3} 5-2x = -1$

Scratchwork:

Need $|f(x)-L| = |5-2x-(-1)| = |6-2x| = |-2(x-3)| = 2|x-3| < \varepsilon.$

$\Rightarrow$ choose $|x-3| < \varepsilon/2.$

Proof: Let $\varepsilon > 0$ be given.

Choose $\delta = \varepsilon/2$

Given x such that $0 < |x-3| < \delta$, then

$|f(x)-L| = |5-2x-(-1)| = |6-2x| = |-2(x-3)|$

$= |-2|\,|x-3| = 2\,|x-3| < \cancel{2} \cdot \varepsilon/\cancel{2} = \varepsilon$ $\square$

3. $f(x) = \dfrac{3-x}{x+7}$

a. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{3-(x+h)}{x+h+7} - \dfrac{3-x}{x+7}}{h}.$$

$$= \lim_{h\to 0}\left(\frac{1}{h}\right) \frac{(3-x-h)(x+7)-(3-x)(x+h+7)}{(x+h+7)(x+7)}$$ common denominator.

$$= \lim_{h\to 0}\left(\frac{1}{h}\right) \frac{\cancel{3x}-\cancel{x^2}-\cancel{xh}+\cancel{21}-\cancel{7x}-7h-(\cancel{3x}+3h+\cancel{21}-\cancel{x^2}-\cancel{xh}-\cancel{7x})}{(x+h+7)(x+7)}$$ distribute algebra.

$$= \lim_{h\to 0}\left(\frac{1}{\cancel{h}}\right) \frac{-10\cancel{h}}{(x+h+7)(x+7)} = \lim_{h\to 0} \frac{-10}{(x+\underset{0}{h}+7)(x+7)} = \boxed{\frac{-10}{(x+7)^2}}$$

b. Quotient Rule.

$$f'(x) = \frac{(x+7)(-1)-(3-x)(1)}{(x+7)^2} = \frac{-\cancel{x}-7-3+\cancel{x}}{(x+7)^2} = \boxed{\frac{-10}{(x+7)^2}}$$

4. $y = x^x$ Logarithmic Differentiation

$\ln y = \ln(x^x)$ Log of both Sides.

$\ln y = x \ln x$ Algebra.

$\dfrac{d}{dx}[\ln y] = \dfrac{d}{dx}[x \ln x]$ Implicitly Differentiate both sides

$$\frac{1}{y}\frac{dy}{dx} = \cancel{x}\left(\frac{1}{\cancel{x}}\right) + \ln x\,(1)$$

$$\frac{dy}{dx} = \overset{x^x}{\cancel{y}}(1+\ln x) = \boxed{x^x(1+\ln x)}$$

5a. $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{1}{u} du$

$= -\ln|u| + C$

$= \boxed{-\ln|\cos x| + C}$

$\boxed{\begin{array}{l} u = \cos x \\ du = -\sin x \, dx \\ -du = \sin x \, dx \end{array}}$

b. $\int_0^{\ln 2} \frac{e^{3x}}{\sqrt{8+e^{3x}}} dx = \frac{1}{3}\int_9^{16} \frac{1}{\sqrt{u}} du = \frac{1}{3} \cdot 2\sqrt{u} \Big|_9^{16}$ ($u^{-1/2}$)

$\boxed{\begin{array}{l} u = 8+e^{3x} \\ du = 3e^{3x} dx \\ \frac{1}{3} du = e^{3x} dx \end{array}}$ $\boxed{\begin{array}{l} x=0 \Rightarrow u = 8+e^{0} = 9 \\ x = \ln 2 \Rightarrow u = 8+e^{3\ln 2} \\ \qquad = 8+e^{\ln 8} = 16 \end{array}}$

$= \frac{2}{3}\left[\sqrt{16} - \sqrt{9}\right] = \boxed{\frac{2}{3}}$

c. $\int_{e^3}^{e^9} \frac{1}{5x} dx = \frac{1}{5}\int_{e^3}^{e^9} \frac{1}{x} dx = \frac{1}{5} \ln|x| \Big|_{e^3}^{e^9}$

$= \frac{1}{5}\left[\ln e^9 - \ln e^3\right] = \boxed{\frac{6}{5}}$

d. $\int_e^{e^4} \frac{3}{x\sqrt{\ln x}} dx = 3\int_1^4 \frac{1}{\sqrt{u}} du = 6\sqrt{u} \Big|_1^4$

$\boxed{\begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array}}$

$= 6\left[\sqrt{4} - \sqrt{1}\right] = \boxed{6}$

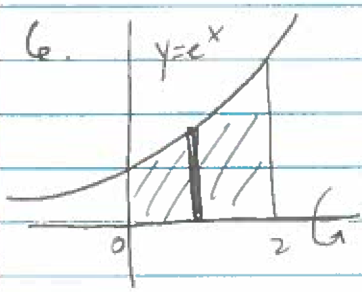
$\boxed{\begin{array}{l} x = e \Rightarrow u = \ln e = 1 \\ x = e^4 \Rightarrow u = \ln e^4 = 4 \end{array}}$

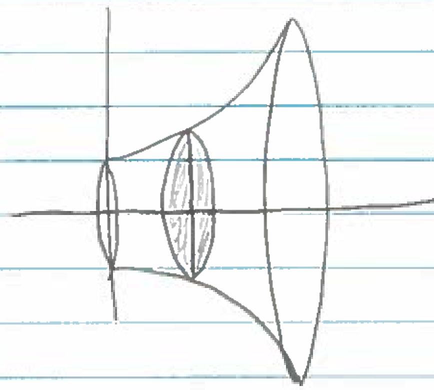
$$5e \quad \int \frac{1}{x(1+\ln x)}\,dx = \int \frac{1}{u}\,du = \ln|u| + C$$

$$= \boxed{\ln\left|1+\ln x\right| + C}$$

$$\boxed{\begin{aligned} u &= 1+\ln x \\ du &= \tfrac{1}{x}\,dx \end{aligned}}$$

6.



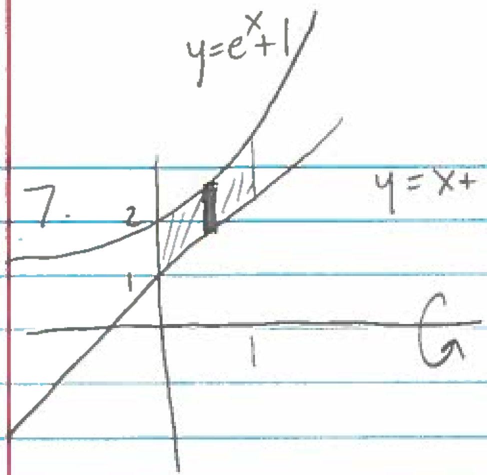


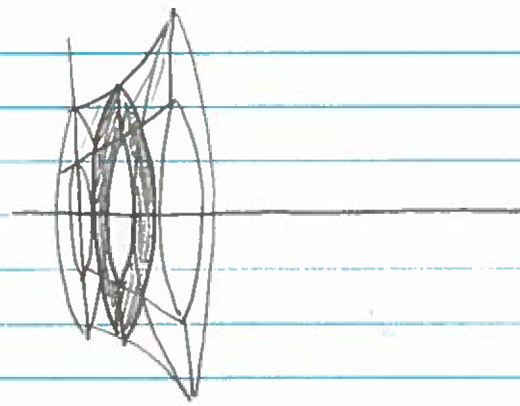
$$V = \pi \int_0^2 (\text{radius})^2\,dx$$

$$= \pi \int_0^2 (e^x)^2\,dx = \pi \int_0^2 e^{2x}\,dx = \pi \left(\frac{e^{2x}}{2}\right)\Big|_0^2$$

$$= \pi\left[\frac{e^4}{2} - \frac{e^{\cancel{0}\,1}}{2}\right] = \boxed{\pi\left[\frac{e^4}{2} - \frac{1}{2}\right]} \text{ or } = \boxed{\frac{\pi}{2}\left(e^4 - 1\right)}$$

7.





$$V = \pi \int_0^1 (\text{outer radius})^2 - (\text{inner radius})^2 \, dx$$

$$= \pi \int_0^1 (e^x+1)^2 - \underbrace{(x+1)^2}_{x^2+2x+1} \, dx$$

$$= \pi \int_0^1 e^{2x} + 2e^x + \cancel{1} - x^2 - 2x - \cancel{1} \, dx$$

$$= \pi \left[\frac{e^{2x}}{2} + 2e^x - \frac{x^3}{3} - x^2 \right] \Bigg|_0^1$$

$$= \pi \left[\frac{e^2}{2} + 2e^1 - \frac{1}{3} - 1 - \left(\frac{e^{0}}{2} + 2e^{0} - 0 - 0 \right) \right]$$

$$= \pi \left[\frac{e^2}{2} + 2e - \frac{1}{3} - 1 - \frac{1}{2} - 2 \right]$$

$-\frac{1}{3} - 3 - \frac{1}{2}$

$-\frac{2}{6} - \frac{18}{6} - \frac{3}{6} = -\frac{23}{6}.$

$$= \boxed{\pi \left[\frac{e^2}{2} + 2e - \frac{23}{6} \right]}$$